MATH 3630
Actuarial Mathematics I
Class Test 1 - 3:35-5:25 PM
Wednesday, 18 September 2019
Time Allowed: 1.5 hours
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____   Student ID: _____

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.

- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.

- Please write legibly.

- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

You are given the following survival function of a newborn:

$$S_0(x) = \left(1 - \frac{x}{105}\right)^{1/5}, \quad \text{for } 0 < x \le 105.$$

Explain why this is a legitimate survival function.

Question No. 2:

You are given:

- The probability that (35) survives to reach age 50 is 0.83.

- The probability that (35) dies between age 50 and 65 is 0.15.

Calculate the probability that (35) survives to reach age 65.

Question No. 3:

Suppose the survival function for a newborn is given by

$$S_0(x) = e^{-0.002x^2}, \quad \text{for } x \geq 0.$$

Calculate μ_{35}.

Question No. 4:

Mortality follows the Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{100}\right)^{2/3}, \text{ for } 0 < x \leq 100.$$

Calculate $_{10|20}q_{20}$ and interpret this probability.

Question No. 5:

For a fixed age x, you are given the following probabilities:

- $p_x = 0.98$

- $p_{x+1} = 0.97$

- $_3p_{x+1} = 0.866$

- $q_{x+3} = 0.06$

Calculate $_3p_x$.

Question No. 6:

In a population consisting of 60% males and 40% females, you are given:

- Mortality for females has a constant force of μ.

- Mortality for males also has a constant force of 3μ, three times that of females.

- Out of the survivors at the end of 10 years, the proportion of males is 50%.

Calculate the probability a female, of any age, survives a year.

Question No. 7:

You are given:

- $\mathring{e}_{30} = 51.50, \quad \mathring{e}_{35} = 46.68, \quad$ and $\quad \mathring{e}_{40} = 41.91$

- $\mathring{e}_{30:\overline{5|}} = 4.988 \quad$ and $\quad \mathring{e}_{30:\overline{10|}} = 9.963$

Calculate $_5p_{35}$.

Question No. 8:

The cumulative distribution function of T_{50}, the future lifetime of (50), is expressed as

$$F_{50}(t) = 1 - \left(1 - \frac{t}{55} \right)^{1/5}, \ \text{ for } 0 < t \le 55.$$

Calculate the probability that (65) dies between age 90 and 100.

Question No. 9:

The force of mortality for a substandard life (x) is expressed as

$$\mu_{x+t}^s = \mu_{x+t} + c,$$

for some constant $c > 0$, where μ_{x+t} is the force of mortality of a standard life (x).

You are given:

- The probability that a standard life (x) survives the next 5 years is 0.75.

- The probability that a substandard life (x) survives the next 5 years is 0.40.

Calculate the value of c.

Question No. 10:

You are given:

$$\mu_x = \begin{cases} 0.05, & 0 < x \le 50 \\ 0.10, & x > 50 \end{cases}$$

Calculate $\mathring{e}_{35:\overline{30|}}$.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK