

3:35

MATH 3630  
Actuarial Mathematics I  
Class Test 1 - 3:55-5:25 PM  
Wednesday, 18 September 2019  
Time Allowed: 1.5 hours  
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: Suggest 1 Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

**Question No. 1:**

You are given the following survival function of a newborn:

$$S_0(x) = \left(1 - \frac{x}{105}\right)^{1/5}, \quad \text{for } 0 < x \leq 105.$$

Explain why this is a legitimate survival function.

Properties to check:

(a)  $S_0(0) = 1$  newborn is alive at 0

$$S_0(0) = \left(1 - \frac{0}{105}\right)^{1/5} = 1$$

(b)  $S_0(\infty) = 0$  all lives eventually die

$$S_0(105) = \left(1 - \frac{105}{105}\right)^{1/5} = 0$$

(c) non-increasing in  $x$

$$\frac{d}{dx} S_0(x) = \frac{1}{5} \left(-\frac{1}{105}\right) \left(1 - \frac{x}{105}\right)^{-4/5} < 0$$

implies non-increasing  
in this case strictly decreasing

## Question No. 2:

You are given:

- The probability that (35) survives to reach age 50 is 0.83.
- The probability that (35) dies between age 50 and 65 is 0.15.

Calculate the probability that (35) survives to reach age 65.



$${}_{15}p_{35} = 0.83$$

~~$${}_{30}p_{35} = 1 - 0.15 = 0.85$$~~

$${}_{15|15}q_{35} = 0.15$$

$${}_{15}p_{35} {}_{15}q_{50} = 0.15$$

$${}_{15}q_{50} = \frac{0.15}{0.83}$$

$${}_{30}p_{35} = {}_{15}p_{35} {}_{15}p_{50}$$

$$= 0.83 \left( 1 - \frac{0.15}{0.83} \right) = 0.83 \left( \frac{.68}{.83} \right)$$

$$= \underline{\underline{0.68}}$$

**Question No. 3:**

Suppose the survival function for a newborn is given by

$$S_0(x) = e^{-0.002x^2}, \quad \text{for } x \geq 0.$$

Calculate  $\mu_{35}$ .

$$\mu_x = -\frac{d}{dx} \log S_0(x) = \frac{d}{dx} (0.002x^2) = 0.004x$$

$$\mu_{35} = 0.004(35) = \underline{\underline{0.14}}$$

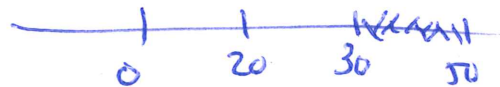
## Question No. 4:

Mortality follows the Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{100}\right)^{2/3}, \text{ for } 0 < x \leq 100.$$

Calculate  ${}_{10|20}q_{20}$  and interpret this probability.

$$\begin{aligned} {}_{10|20}q_{20} &= \frac{S_0(30) - S_0(50)}{S_0(20)} \\ &= \frac{\left(\frac{70}{100}\right)^{2/3} - \left(\frac{50}{100}\right)^{2/3}}{\left(\frac{80}{100}\right)^{2/3}} \\ &= \frac{70^{2/3} - 50^{2/3}}{80^{2/3}} \approx \underline{0.183822} \end{aligned}$$



This gives the probability that life (20) will die between ages 30 and 50.

This is equivalent to  $S_{20}(10) - S_{20}(30) = \left(\frac{70}{80}\right)^{2/3} - \left(\frac{50}{80}\right)^{2/3}$

where  $S_{20}(t) = \left(1 - \frac{t}{80}\right)^{2/3}$

since  $T_{20} \sim \text{GDM}(\alpha = 2/3, \omega - 20 = 80)$

## Question No. 5:

For a fixed age  $x$ , you are given the following probabilities:

- $p_x = 0.98$
- $p_{x+1} = 0.97$
- ${}_3p_{x+1} = 0.866$
- $q_{x+3} = 0.06$

Calculate  ${}_3p_x$ .

This is similar to Exercise 2.6 of book!

$${}_3p_x = p_x \cdot p_{x+1} \cdot p_{x+2}$$

$${}_3p_{x+1} = p_{x+1} \cdot p_{x+2} \cdot p_{x+3} \Rightarrow p_{x+1} \cdot p_{x+2} = \frac{{}_3p_{x+1}}{p_{x+3}} = \frac{.866}{1-.06}$$

$$\frac{.866}{.94}$$

Therefore,

$${}_3p_x = 0.98 * \frac{.866}{.94}$$

$$= \underline{\underline{0.908511}}$$

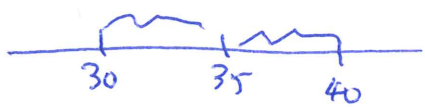
## Question No. 7:

You are given:

- $\ddot{e}_{30} = 51.50$ ,  $\ddot{e}_{35} = 46.68$ , and  $\ddot{e}_{40} = 41.91$

- $\ddot{e}_{30:\overline{5}|} = 4.988$  and  $\ddot{e}_{30:\overline{10}|} = 9.963$

Calculate  ${}_5p_{35}$ .

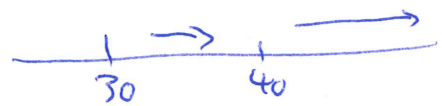


$$\ddot{e}_{30} = \ddot{e}_{30:\overline{5}|} + {}_5p_{30} \ddot{e}_{35} \Rightarrow {}_5p_{30} = \frac{\ddot{e}_{30} - \ddot{e}_{30:\overline{5}|}}{\ddot{e}_{35}}$$

$\begin{array}{ccc} 51.50 & & 4.988 \\ \uparrow & & \uparrow \\ \ddot{e}_{30} & - & \ddot{e}_{30:\overline{5}|} \\ \hline & & \ddot{e}_{35} \quad 46.68 \end{array}$

$$= 0.996401$$

Similarly, we get



$$\ddot{e}_{30} = \ddot{e}_{30:\overline{10}|} + {}_{10}p_{30} \ddot{e}_{40}$$

$$\Rightarrow {}_{10}p_{30} = \frac{51.50 - 9.963}{41.91} = 0.9911$$

Thus, we have

$${}_{10}p_{30} = {}_5p_{30} \cdot {}_5p_{35} \Rightarrow {}_5p_{35} = \frac{{}_{10}p_{30}}{{}_5p_{30}} = \frac{0.9911}{0.996401}$$

$$= \underline{\underline{0.9946798}}$$



## Question No. 8:

The cumulative distribution function of  $T_{50}$ , the future lifetime of (50), is expressed as

$$F_{50}(t) = 1 - \left(1 - \frac{t}{55}\right)^{1/5}, \text{ for } 0 < t \leq 55.$$

Calculate the probability that (65) dies between age 90 and 100.

$$\begin{aligned}
 {}_{25/10}p_{65} &= \frac{S_{65}(25) - S_{65}(35)}{\cancel{S_{65}(65)}} && \begin{array}{c} 25 \quad 10 \\ \text{---} \quad \text{---} \\ 65 \quad 90 \quad 100 \end{array} \\
 &= \frac{S_0(90)}{S_0(65)} - \frac{S_0(100)}{S_0(65)} && \begin{array}{l} \text{divide and multiply} \\ \text{by } S_0(50) \end{array} \\
 &= \frac{S_0(90)/S_0(50) - S_0(100)/S_0(50)}{S_0(65)/S_0(50)} \\
 &= \frac{S_{50}(40) - S_{50}(50)}{S_{50}(15)} \\
 &= \frac{\left(\frac{15}{55}\right)^{1/5} - \left(\frac{5}{55}\right)^{1/5}}{\left(\frac{40}{55}\right)^{1/5}} \\
 &= \frac{15^{1/5} - 5^{1/5}}{40^{1/5}} = \underline{\underline{0.162122}}
 \end{aligned}$$



**Question No. 9:**

The force of mortality for a substandard life ( $x$ ) is expressed as

$$\mu_{x+t}^s = \mu_{x+t} + c,$$

for some constant  $c > 0$ , where  $\mu_{x+t}$  is the force of mortality of a standard life ( $x$ ).

You are given:

- The probability that a standard life ( $x$ ) survives the next 5 years is 0.75.
- The probability that a substandard life ( $x$ ) survives the next 5 years is 0.40.

Calculate the value of  $c$ .

For a standard life,  ${}_5p_x = 0.75 = e^{-\int_0^5 \mu_{x+s} ds}$

For a substandard life,

$${}_5p_x^s = e^{-\int_0^5 (\mu_{x+s} + c) ds}$$

$$= \underbrace{e^{-\int_0^5 \mu_{x+s} ds}}_{{}_5p_x} \cdot e^{-5c} = 0.75 e^{-5c} = 0.40$$

Solving for  $c$ , we set

$$\begin{aligned} e^{-5c} &= \frac{0.40}{0.75} = \frac{40}{75} = \frac{8}{15} \Rightarrow c = -\frac{1}{5} \log\left(\frac{8}{15}\right) \\ &= \log\left(\frac{8}{15}\right)^{-1/5} \\ &= 0.1257217 \\ &\approx \boxed{0.13} \end{aligned}$$

## Question No. 10:

You are given:

$$\mu_x = \begin{cases} 0.05, & 0 < x \leq 50 \\ 0.10, & x \geq 50 \end{cases}$$

Calculate  $\ddot{e}_{35:\overline{30}|}$ .

or  $0 < t \leq 15$   
 For  $0 < x \leq 50$ , we have

$${}_t p_{35} = e^{-0.05t}, \quad 0 < t \leq 15$$

For  $x > 50$ , we have  
 or  $t > 15$

$${}_t p_{35} = {}_{15} p_{35} \cdot {}_{t-15} p_{50}$$

$$= e^{-0.05(15)} e^{-0.10(t-15)}, \quad t > 15$$

$$= e^{-0.05(15) - 0.10t}, \quad t > 15$$

$$\ddot{e}_{35:\overline{30}|} = \int_0^{30} {}_t p_{35} dt$$

$$= \int_0^{15} e^{-0.05t} dt + \int_{15}^{30} e^{-0.05(15)} e^{-0.10(t-15)} dt$$

$$e^{-0.75} \int_0^{15} e^{-0.10s} ds$$

$$= \frac{1 - e^{-0.75}}{0.05} + e^{-0.75} \frac{1 - e^{-1.5}}{0.10} = \underline{\underline{14.22234}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK