MATH 3630
Actuarial Mathematics I
Class Test 1 - 3:55-5:25 PM
Wednesday, 26 September 2018
Time Allowed: 1.5 hours
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: _____ **Student ID:** _____

- **There are twelve (12) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

You are given the following survival function of a newborn:

$$S_0(x) = \exp\left[-(x/8)^{1/2}\right], \quad \text{for } x \geq 0.$$

Calculate ${}_{15|20}q_{25}$ and interpret this probability.

Question No. 2:

You are given:

- ${}_5p_{40} = 0.95$
- ${}_5p_{45} = 0.90$
- $\ell_{40} = 10,000$

Calculate ${}_5d_{45}$.

Question No. 3:

In a population of newborn consisting of 75% males and 25% females, you are given:

- Mortality for males follow De Moivre's law with $\omega = 90$.
- Mortality for females follow De Moivre's law with $\omega = 120$.

At what age will there be exactly equal proportions of male and female?

Question No. 4:

Let X be the age-at-death random variable with

$$\mu_x = \frac{1}{3(110-x)}, \quad \text{for } 0 \leq x < 110.$$

Find an expression for $f_{40}(t)$, the density function of future lifetime of (40).

Question No. 5:

For a life (x), you are given the following extract from a life table:

k	ℓ_{x+k}
0	10,000
1	9,900
2	9,700
3	9,400
4	9,000
5	8,500

Calculate the probability that $(x+1)$ will die between ages $x+4$ and $x+5$.

Question No. 6:

Suppose you are given the following extract from a life table:

x	ℓ_x
95	15,000
96	12,500
97	9,000
98	4,500
99	1,500
100	100
101	0

Calculate e_{96}.

Question No. 7:

You are given:

$$ {}_{k|}q_x = \frac{1}{9}(2k+1), \quad \text{for} \quad k = 0, 1 \text{ and } 2. $$

Suppose Uniform Distribution of Death (UDD) holds between integral ages.

Calculate the probability that a life (x) will survive another 1.5 years.

Question No. 8:

You are given:

- The probability that (40) will live for the next 10 years is 0.80.
- The probability that (40) dies between ages 50 and 75 is 0.25.
- $\ell_{75} = 7,200$

Calculate ℓ_{50}.

Question No. 9:

The force of mortality for a substandard life (x) is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + a,$$

for some constant $a > 0$, where μ_{x+t} is the force of mortality of a standard life (x).
You are given:

- The probability that a standard life (x) survives the next 10 years is 0.70.
- The probability that a substandard life (x) survives the next 10 years is 0.63.

Calculate the value of the constant a.

Question No. 10:

You are given:

$$\ell_{90} = 10,000 \qquad \ell_{91} = 9,000 \qquad \ell_{92} = 7,500 \qquad \ell_{93} = 5,500$$

Suppose that constant force of mortality assumption holds between integral ages.

Calculate ${}_{1.5|0.75}q_{90}$.

Question No. 11:

You are given:

- $\mathring{e}_{30} = 51.50, \quad \mathring{e}_{35} = 46.68, \quad \text{and} \quad \mathring{e}_{40} = 41.91$
- $\mathring{e}_{30:\overline{5}|} = 4.988 \quad \text{and} \quad \mathring{e}_{30:\overline{10}|} = 9.963$

Calculate ${}_5p_{35}$.

Question No. 12:

For a life (x), you are given $\ell_x = 100,000$ and the following extract from a life table:

k	d_{x+k}
0	1,000
1	1,500
2	2,100
3	3,000
4	2,000

Calculate ${}_{2|}q_{x+1}$.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK