MATH 3630
Actuarial Mathematics I
Class Test 1 - 3:55-5:25 PM
Wednesday, 26 September 2018
Time Allowed: 1.5 hours
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: Emil Student ID: Suggested Solutions

- There are twelve (12) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

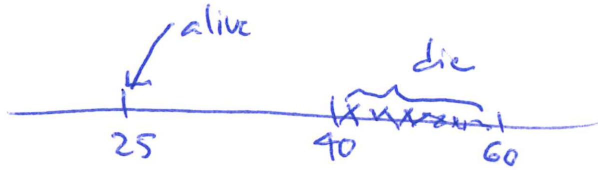
Question No. 1:

You are given the following survival function of a newborn:

$$S_0(x) = \exp\left[-(x/8)^{1/2}\right], \quad \text{for } x \geq 0.$$

Calculate ${}_{15|20}q_{25}$ and interpret this probability.

refers to the probability that a 25-year-old will die between ages 40 and 60



$${}_{15|10}q_{25} = \frac{S_0(40) - S_0(60)}{S_0(25)}$$

$$= \frac{e^{-(40/8)^{1/2}} - e^{-(60/8)^{1/2}}}{e^{-(25/8)^{1/2}}}$$

$$= \underline{\underline{0.2473025}}$$

Question No. 2:

You are given:

- ${}_5p_{40} = 0.95$
- ${}_5p_{45} = 0.90$
- $\ell_{40} = 10,000$

Calculate ${}_5d_{45}$.

$\ell_{45} = 0.95\, \ell_{40} = 9,500$

${}_5d_{45} = {}_5q_{45} \cdot \ell_{45} = (1 - {}_5p_{45}) \cdot \ell_{45}$

$= (1 - 0.90) \cdot 9500$

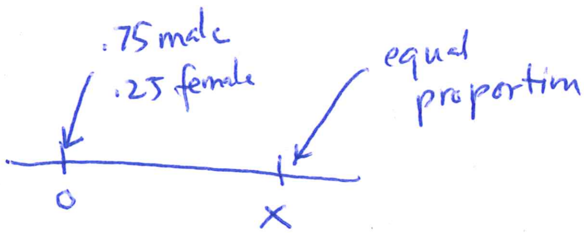
$= \underline{\underline{950}}$

Question No. 3:

In a population of newborn consisting of 75% males and 25% females, you are given:

- Mortality for males follow De Moivre's law with $\omega = 90$.
- Mortality for females follow De Moivre's law with $\omega = 120$.

At what age will there be exactly equal proportions of male and female?



Let x be the age when there are exactly equal proportions

proportion of females at age x is

$$\underbrace{\frac{0.25 * \left(1 - \frac{x}{120}\right)}{0.25\left(1 - \frac{x}{120}\right) + 0.75\left(1 - \frac{x}{90}\right)}} = \frac{120 - x}{(120 - x) + \cancel{3}\left(\frac{\cancel{120}^{4}}{\cancel{90}_{3}}\right)(90 - x)}$$

Simplifying we get

$$\frac{120 - x}{480 - 5x} = 0.5 \Rightarrow \underline{\underline{x = 80}}$$

Question No. 4:

Let X be the age-at-death random variable with

$$\mu_x = \frac{1}{3(110-x)}, \quad \text{for } 0 \le x < 110.$$

Find an expression for $f_{40}(t)$, the density function of future lifetime of (40).

$$\mu_{40+t} = \frac{1}{3(70-t)}, \quad 0 \le t < 70$$

$${}_tp_{40} = e^{-\int_0^t \mu_{40+s}\,ds}$$

$$= e^{-\frac{1}{3}\int_0^t \frac{1}{70-s}\,ds} = e^{\frac{1}{3}\log(70-s)\Big|_0^t} = e^{\frac{1}{3}\log(1-t/70)}$$

$$= (1-t/70)^{1/3}$$

$$f_{40}(t) = {}_tp_{40}\,\mu_{40+t}$$

$$= \frac{1}{3}\,\frac{(70-t)^{1/3}/70^{1/3}}{(70-t)}$$

$$= \frac{1}{3(70)^{1/3}}(70-t)^{-2/3}, \quad 0 \le t < 70$$

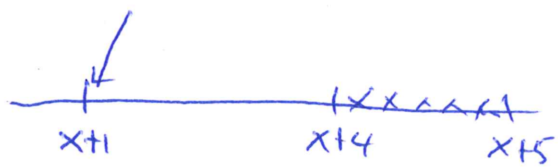
Check if $\int_0^{70} f_{40}(t)\,dt = 1$!

Question No. 5:

For a life (x), you are given the following extract from a life table:

k	ℓ_{x+k}
0	10,000
1	9,900
2	9,700
3	9,400
4	9,000
5	8,500

Calculate the probability that $(x+1)$ will die between ages $x+4$ and $x+5$.



$$\text{probability} = {}_{3|}q_{x+1}$$

$$= \frac{l_{x+4} - l_{x+5}}{l_{x+1}} = \frac{9000 - 8500}{9900}$$

$$= \frac{5\cancel{00}}{99\cancel{00}}$$

$$= \frac{5}{99} \approx 0.050505$$

Question No. 6:

Suppose you are given the following extract from a life table:

x	ℓ_x
95	15,000
96	12,500
97	9,000
98	4,500
99	1,500
100	100
101	0

$$e_x = \sum_{k=1}^{\infty} {}_kp_x$$

Calculate e_{96}.

$$e_{96} = \sum_{k=1}^{\infty} {}_kp_{96} = \sum_{k=1}^{\infty} \ell_{96+k}/\ell_{96}$$

$$= \frac{1}{12500}(9000 + 4500 + 1500 + 100)$$

$$= \frac{15100}{12500} = \underline{\underline{1.208}}$$

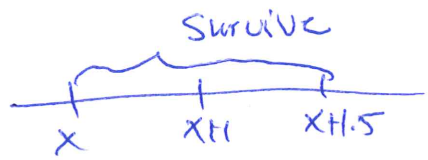
Question No. 7:

You are given:

$$_{k|}q_x = \frac{1}{9}(2k+1), \quad \text{for} \quad k = 0, 1 \text{ and } 2.$$

Suppose Uniform Distribution of Death (UDD) holds between integral ages.

Calculate the probability that a life (x) will survive another 1.5 years.



$$_{0|}q_x = q_x = \frac{1}{9}$$

$$_{1|}q_x = p_x\, q_{x+1} = \frac{8}{9} q_{x+1} = \frac{1}{9}(2+1) = \frac{3}{9} \Rightarrow q_{x+1} = \frac{3}{8}$$

Therefore, we have

$$_{1.5}p_x = p_x \times {}_{.5}p_{x+1}$$

$$= p_x \times (1 - 0.5\, q_{x+1})$$

applying UDD assumption $_{k}q_x = k \cdot q_x$

$$= \frac{8}{9}\left(1 - 0.5 \frac{3}{8}\right)$$

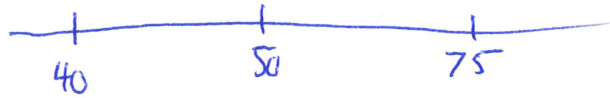
$$= \frac{\cancel{8}}{9} \times \frac{13}{\cancel{16}_{2}} = \frac{13}{18} \approx \underline{\underline{0.7222}}$$

Question No. 8:

You are given:

- The probability that (40) will live for the next 10 years is 0.80.
- The probability that (40) dies between ages 50 and 75 is 0.25.
- $\ell_{75} = 7,200$

Calculate ℓ_{50}.



$${}_{10}p_{40} = \frac{l_{50}}{l_{40}} = 0.80 \Rightarrow l_{50} = 0.80\, l_{40}$$

$${}_{10|25}q_{50} = \frac{l_{50} - l_{75}}{l_{40}} = \frac{0.80\, l_{40} - 7200}{l_{40}} = 0.25$$

$$0.55\, l_{40} = 7200$$

$$l_{40} = \frac{7200}{0.55} = 13090.91$$

$$\therefore\ l_{50} = 0.80\,(13090.91)$$

$$= 10,472.73 \approx \underline{\underline{10,473}}$$

Question No. 9:

The force of mortality for a substandard life (x) is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + a,$$

for some constant $a > 0$, where μ_{x+t} is the force of mortality of a standard life (x). You are given:

- The probability that a standard life (x) survives the next 10 years is 0.70.
- The probability that a substandard life (x) survives the next 10 years is 0.63.

Calculate the value of the constant a.

standard life (x): $\quad {}_{10}p_x = 0.70$

Substandard life (x): $\quad {}_{10}p_x^s = e^{-\int_0^{10} \mu^s_{x+t} dt} = \underbrace{e^{-\int_0^{10} \mu_{x+t} dt}}_{{}_{10}p_x} e^{-10a}$

$$\doteq 0.70 e^{-10a} = 0.63$$

Solving for a, we set

$$e^{-10a} = 0.63/0.70$$

$$-10a = \log(0.63/0.70)$$

$$a = -\frac{1}{10} \log(0.63/0.70)$$

$$= 0.0105360S \approx \underline{\underline{0.011}}$$

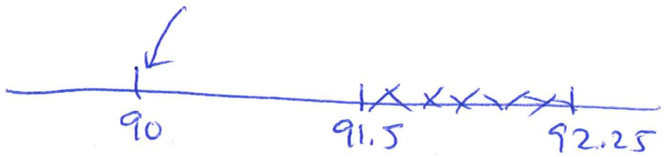
Question No. 10:

You are given:

$$\ell_{90} = 10,000 \qquad \ell_{91} = 9,000 \qquad \ell_{92} = 7,500 \qquad \ell_{93} = 5,500$$

Suppose that constant force of mortality assumption holds between integral ages.

Calculate ${}_{1.5|0.75}q_{90}$.



$${}_{1.5|0.75}q_{90} = \frac{l_{91.5} - l_{92.25}}{l_{90}}$$

$$= \frac{l_{91}^{.5}\, l_{92}^{.5} - l_{92}^{.75}\, l_{93}^{.25}}{l_{90}}$$

$$= \frac{(9000)^{.5}(7500)^{.5} - (7500)^{.75}(5500)^{.25}}{10000}$$

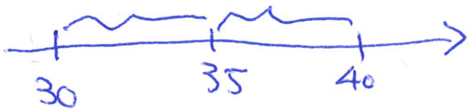
$$= \underline{\underline{0.1275405}}$$

Question No. 11:

You are given:

- $\mathring{e}_{30} = 51.50$, $\mathring{e}_{35} = 46.68$, and $\mathring{e}_{40} = 41.91$
- $\mathring{e}_{30:\overline{5}|} = 4.988$ and $\mathring{e}_{30:\overline{10}|} = 9.963$

Calculate ${}_5p_{35}$.



$$\mathring{e}_{30} = \mathring{e}_{30:\overline{5}|} + {}_5p_{30}\,\mathring{e}_{35} \Rightarrow {}_5p_{30} = \frac{\overset{51.50}{\mathring{e}_{30}} - \overset{4.988}{\mathring{e}_{30:\overline{5}|}}}{\underset{46.68}{\mathring{e}_{35}}}$$

$$= 0.996401$$

Similarly, we get $\mathring{e}_{30} = \mathring{e}_{30:\overline{10}|} + {}_{10}p_{30}\,\mathring{e}_{40}$

$$\Rightarrow {}_{10}p_{30} = \frac{\mathring{e}_{30} - \mathring{e}_{30:\overline{10}|}}{\mathring{e}_{40}} = \frac{51.50 - 9.963}{41.91} = 0.9911$$

$${}_{10}p_{30} = {}_5p_{30}\;{}_5p_{35} \Rightarrow {}_5p_{35} = {}_{10}p_{30} / {}_5p_{30}$$

$$= 0.9911 / 0.996401$$

$$= \underline{\underline{0.994679 8}}$$

Question No. 12:

For a life (x), you are given $\ell_x = 100,000$ and the following extract from a life table:

k	d_{x+k}
0	1,000
1	1,500
2	2,100
3	3,000
4	2,000

$$l_x - l_{x+1} = d_x$$

$$l_{x+1} = l_x - d_x$$

$$= 100000 - 1000$$

$$= 99,000$$

Calculate ${}_{2|}q_{x+1}$.

$x+1$ $x+3$ $x+4$

$${}_{2|}q_{x+1} = \frac{d_{x+3}}{l_{x+1}}$$

$$= \frac{3\cancel{000}}{99\cancel{000}} = \frac{1}{33} \approx \underline{\underline{0.0303}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK