MATH 3630

Actuarial Mathematics I

Class Test 1 - 3:35-4:50 PM

Wednesday, 27 September 2017

Time Allowed: 1 hour

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: Suggested Solutions

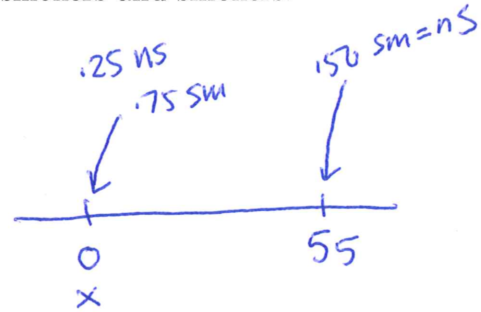
- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

In a population today, all of equal age x, consisting of 25% non-smokers and 75% smokers, you are given:

- Mortality for non-smokers follows a constant force of mortality of 0.01.
- Mortality for smokers follows a constant force of mortality of $0.01h$, for some positive h.
- In 55 years, there will exactly be equal proportions of non-smokers and smokers.

Calculate h.



proportion of non-smokers in 55 years

$$\frac{0.25 e^{-.01(55)}}{0.25 e^{-.01(55)} + 0.75 e^{-.01h(55)}}$$

$$= \frac{\cancel{0.25 e^{-.01(55)}}}{\cancel{0.25 e^{-.01(55)}}\left(1+3e^{-.55(h-1)}\right)} = \frac{1}{2}$$

$$\Rightarrow \quad 2 = 1 + 3e^{-.55(h-1)}$$

$$\log\frac{1}{3} = -.55(h-1)$$

$$\frac{\log 3}{.55} + 1 = h$$

$\Rightarrow \quad h = 2.997477 \approx 3$ times that of non-smokers.

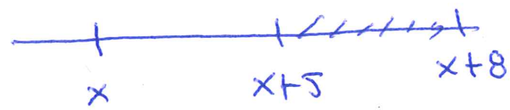
Question No. 2:

You are given:

- ${}_5p_x = 0.96$
- ${}_8p_x = 0.90$

Calculate ${}_3q_{x+5}$.

$${}_8p_x = {}_5p_x * {}_3p_{x+5}$$



$${}_3q_{x+5} = 1 - {}_3p_{x+5}$$

$$= 1 - \frac{{}_8p_x}{{}_5p_x} = 1 - \frac{.90}{.96} = \frac{.06}{.96} = \frac{6}{96} = \frac{1}{16}$$

$$= 0.0625$$

Question No. 3:

You are given the following survival function of a newborn:

$$S_0(x) = \exp\left[-(2x/15)^{3/4}\right], \quad \text{for } x \geq 0.$$

Calculate the force of mortality at age 45, μ_{45}.

$$\mu_x = \frac{-d}{dx} \log S_0(x) = \frac{d}{dx}\left[\left(2x/15\right)^{3/4} \right]$$

$$= \frac{\cancel{3}}{\cancel{4}_{2}}\left(\frac{2x}{15}\right)^{-1/4}\left(\frac{\cancel{2}}{\cancel{15}_{5}}\right)$$

$$= \frac{1}{10}\left(\frac{2x}{15}\right)^{-1/4}$$

at $x = 45$,

$$\mu_{45} = \frac{1}{10}\left(\frac{2(\cancel{45}^{3})}{\cancel{15}}\right)^{-1/4} = \frac{1}{10} 6^{-1/4} = \underline{\underline{0.0638943}}$$

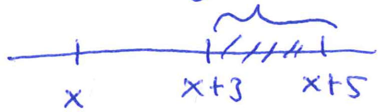
Question No. 4:

For a life (x), you are given the following extract from a life table:

k	ℓ_{x+k}
0	10,000
1	9,875
2	9,625
3	9,275
4	8,775
5	8,025

Calculate ${}_{3|2}q_x$ and interpret this probability.

# of deaths $= l_{x+3} - l_{x+5}$



This is the probability that a life (x) will die between ages x+3 and x+5

$${}_{3|2}q_x = \frac{l_{x+3} - l_{x+5}}{l_x} = \frac{9275 - 8025}{10000}$$

$$= \underline{\underline{0.125}}$$

Question No. 5:

You are given:

- Mortality follows De Moivre's law with parameter ω.
- $\mathring{e}_{20:\overline{20}|} = 18$

Calculate ${}_{30|10}q_{30}$.

$T_{20} \sim$ Uniform on 0 and $\omega - 20$

$$\mathring{e}_{20:\overline{20}|} = \int_0^{20} \overbrace{\left(1 - \frac{1}{\omega-20} t\right)}^{{}_tp_{20}} dt \qquad {}_tp_{20} = 1 - \frac{t}{\omega - 20}$$

$$= 20 - \frac{1}{\omega-20} \frac{1}{2} 20^2$$

$$= 20 - \frac{1}{\omega - 20} 200 = 18$$

Thus, $2 = \frac{200}{\omega-20}$ or $\omega - 20 = 100$

$\omega = 120$ years

$${}_{30|10}q_{30} = \frac{10}{90} = \frac{1}{9}$$

$$= \underline{\underline{0.1111}}$$

$T_{30} \sim$ Uniform on 0 and 90

Question No. 6:

Suppose you are given the following extract from a life table:

x	ℓ_x
94	15,000
95	12,500
96	8,750
97	4,375
98	1,530
99	380
100	40
101	0

Calculate e_{95}.

$$e_{95} = \sum_{k=1}^{\infty} {}_kp_{95}$$

$$= \frac{l_{96} + l_{97} + l_{98} + l_{99} + l_{100}}{l_{95}}$$

$$= \frac{8750 + 4375 + 1530 + 380 + 40}{12500}$$

$$= \frac{15075}{12500}$$

$$= 1.206$$

Question No. 7:

You are given:

- The probability that (35) survives to reach age 50 is 0.83.
- The probability that (35) dies between the ages of 50 and 65 is 0.15.
- $\ell_{65} = 6800$

Calculate ℓ_{35}.

$${}_{15}p_{35} = 0.83$$

$${}_{15|15}q_{35} = 0.15$$

$${}_{15}p_{35} \cdot (1 - {}_{15}p_{50}) = 0.15$$

$${}_{15}p_{35} = 0.83$$

$${}_{15}p_{50} = 1 - \frac{.15}{.83} = \frac{68}{83}$$

$${}_{30}p_{35} = \frac{l_{65}}{l_{35}}$$

$$= {}_{15}p_{35} \cdot {}_{15}p_{50}$$

$$= 0.83\left(\frac{68}{83}\right) = 0.68$$

$$l_{35} = \frac{l_{65}}{0.68} = \underline{\underline{10{,}000}}$$

Question No. 8:

Mortality follows the Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{100}\right)^{1/2}, \text{ for } 0 \leq x \leq 100.$$

Calculate the probability that life (35) will die between ages 50 and 65.

35 50 65

$$_{15|15}q_{50} = \frac{S_0(50) - S_0(65)}{S_0(35)}$$

$$= \frac{(1/2)^{.5} - (.35)^{.5}}{(.65)^{.5}}$$

$$= \frac{0.1154988}{0.8062258}$$

$$= \underline{\underline{0.1432586}}$$

Question No. 9:

The force of mortality for a substandard life (x) is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + a,$$

for some constant $a > 0$, where μ_{x+t} is the force of mortality of a standard life (x). You are given:

- The probability that a standard life (x) survives the next 10 years is 0.70.
- The probability that a substandard life (x) survives the next 10 years is 0.63.

Calculate the value of the constant a.

standard life: $\quad {}_{10}p_x = e^{-\int_0^{10} \mu_{x+t}\,dt} = 0.70$

substandard life: $\quad {}_{10}p_x^s = e^{-\int_0^{10} (\mu_{x+t}+a)\,dt}$

$$= \underbrace{e^{-\int_0^{10} \mu_{x+t}\,dt}}_{{}_{10}p_x = 0.70} \; e^{-10a} = 0.63$$

$$\Rightarrow e^{-10a} = \frac{63}{70} = 0.90$$

$$\Rightarrow -10a = \log(0.90)$$

$$\Rightarrow a = \frac{\log(0.90)}{-10} = 0.01053605$$

Question No. 10:

Please complete the rest of the life table below:

x	ℓ_x	d_x	p_x	q_x
96	100	100-85 = 15	$\frac{85}{100}$ = .850	1-.850 = 0.150
97	85	85-65 = 20	$\frac{65}{85}$ = 0.765	1-0.765 = 0.235
98	65	65-35 = 30	$\frac{35}{65}$ = 0.538	1-0.538 = 0.462
99	35	35	$\frac{0}{35}$ = 0	1-0 = 1.000
100	0	na	na	na

na = not applicable

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK