**MATH 3630**
**Actuarial Mathematics I**
**Class Test 1 - 3:35-4:50 PM**
**Wednesday, 28 September 2016**
**Time Allowed: 1 hour**
**Total Marks: 100 points**

**Please write your name and student number at the spaces provided:**

**Name:** _____ **Student ID:** _____

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

**Question No. 1:**

You are given:

$$\mu_x = \begin{cases} 0.01, & 0 < x < 30 \\ 0.02, & x \geq 30 \end{cases}$$

Calculate the probability that a 25-year-old will survive another 25 years.

**Question No. 2:**

In a population of newborn consisting of 70% males and 30% females, you are given:

- Mortality for males follow De Moivre's law with $\omega = 100$.
- Mortality for females follow De Moivre's law with $\omega = 120$.

At what age will there be exactly equal proportions of male and female?

**Question No. 3:**

For a life $(x)$, you are given $\ell_x = 10,000$ and the following extract from a life table:

| $k$ | $d_{x+k}$ |
|---|---|
| 0 | 125 |
| 1 | 250 |
| 2 | 350 |
| 3 | 500 |
| 4 | 750 |
| 5 | 985 |

Calculate ${}_{3|}q_{x+1}$ and interpret this probability.

**Question No. 4:**

An organism has a very short lifetime with its mortality described by the force of mortality

$$\mu_x = \frac{2}{1+x}, \quad \text{for } x \geq 0$$

Calculate the probability that such an organism who has lived one year will survive another two years.

**Question No. 5:**

You are given:

- The probability that (30) survives to reach age 50 is 0.75.
- The probability that (30) dies between the ages of 50 and 65 is 0.15.
- $\ell_{30} = 1000$

Calculate $\ell_{65}$.

**Question No. 6:**

Mortality of a newborn follows the Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{100}\right)^{2/5}, \text{ for } 0 \leq x \leq 100.$$

Calculate the median lifetime of a 50-year-old.

**Question No. 7:**

Suppose you are given the following extract from a life table:

| $x$ | $\ell_x$ |
|---|---|
| 94 | 15,000 |
| 95 | 12,500 |
| 96 | 8,750 |
| 97 | 4,375 |
| 98 | 1,530 |
| 99 | 380 |
| 100 | 40 |
| 101 | 0 |

Calculate ${}_{2|3}q_{95}$.

**Question No. 8:**

Suppose you are given:

$$\mu_x = \begin{cases} 0.02, & 0 < x < 35 \\ 0.04, & x \geq 35 \end{cases}$$

Calculate $\mathring{e}_{25:\overline{25|}}$.

**Question No. 9:**

For a fixed age $x$, you are given the following probabilities:

- $p_x = 0.95$
- ${}_3q_x = 0.24$

Calculate the probability that $(x+1)$ will not survive the following two years.

**Question No. 10:**

The force of mortality for a substandard life $(x)$ is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + a,$$

for some constant $a > 0$, where $\mu_{x+t}$ is the force of mortality of a standard life $(x)$.
You are given:

- The probability that a standard life $(x)$ survives the next 20 years is 0.50.
- The probability that a substandard life $(x)$ survives the next 20 years is 0.25.

Calculate the value of the constant $a$.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK