MATH 3630
Actuarial Mathematics I
Class Test 1 - 3:35-4:50 PM
Wednesday, 28 September 2016
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL

Student ID: Suggested Solutions

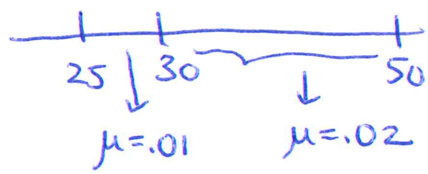
- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

You are given:

$$\mu_x = \begin{cases} 0.01, & 0 < x < 30 \\ 0.02, & x \geq 30 \end{cases}$$

Calculate the probability that a 25-year-old will survive another 25 years.



$$ {}_{25}p_{25} = {}_{5}p_{25} * {}_{20}p_{30} $$

$$ = e^{-.01(5)} * e^{-.02(20)} $$

$$ = e^{-.45} $$

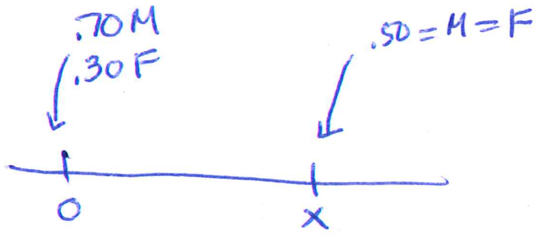
$$ = \underline{\underline{0.6376282}} $$

Question No. 2:

In a population of newborn consisting of 70% males and 30% females, you are given:

- Mortality for males follow De Moivre's law with $\omega = 100$.
- Mortality for females follow De Moivre's law with $\omega = 120$.

At what age will there ~~will~~ be exactly equal proportions of male and female?



proportion of females at age x

$$\frac{0.30 * \left(1 - \frac{x}{120}\right)}{0.70\left(1 - \frac{x}{100}\right) + 0.30\left(1 - \frac{x}{120}\right)}$$

$$\frac{0.30(120 - x)}{0.70\left(\frac{120}{100}\right)(100 - x) + 0.30(120 - x)}$$

Simplifying further we get

$$\frac{36 - .30x}{120 - 1.14x} = 50\% \Rightarrow \underline{\underline{x = 88.88889}}$$

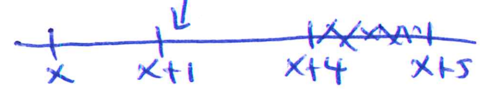
Question No. 3:

For a life (x), you are given $\ell_x = 10,000$ and the following extract from a life table:

k	d_{x+k}
0	125
1	250
2	350
3	500
4	750
5	985

Calculate ${}_{3|}q_{x+1}$ and interpret this probability.

$l_{x+1} = 10000 - 125 = 9875$



$d_{x+4} = 750$

d_{x+4}

$${}_{3|}q_{x+1} = \frac{750}{9875} = \underline{\underline{0.07594937}}$$

l_{x+1}

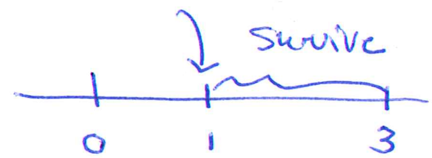
This is the probability that a life $(x+1)$ will die between ages $x+4$ and $x+5$.

Question No. 4:

An organism has a very short lifetime with its mortality described by the force of mortality

$$\mu_x = \frac{2}{1+x}, \quad \text{for } x \geq 0$$

Calculate the probability that such an organism who has lived one year will survive another two years.



$${}_2p_1 = e^{-\int_0^2 \mu_{s+1} ds}$$

$$= e^{-\int_0^2 \frac{2}{2+s} ds}$$

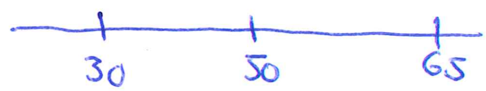
$$= e^{-2\log\left(\frac{4}{2}\right)} = \left(\frac{4}{2}\right)^{-2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = \underline{\underline{0.25}}$$

Question No. 5:

You are given:

- The probability that (30) survives to reach age 50 is 0.75.
- The probability that (30) dies between the ages of 50 and 65 is 0.15.
- $\ell_{30} = 1000$

Calculate ℓ_{65}.



$${}_{20}p_{30} = 0.75$$

$${}_{20|15}q_{30} = 0.15 \quad \Rightarrow \quad {}_{20}p_{30} \cdot {}_{15}q_{50} = 0.75 \; {}_{15}q_{50} = 0.15$$

$$\Rightarrow \quad {}_{15}q_{50} = \frac{0.15}{0.75} = .2$$

$${}_{35}p_{30} = {}_{20}p_{30} * {}_{15}p_{50}$$

$$= 0.75 * 0.80 = 0.60 = \frac{l_{65}}{l_{30}}$$

$$l_{65} = 0.60 \, l_{30} = 0.60(1000) = \underline{\underline{600}}$$

Question No. 6:

Mortality of a newborn follows the Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{100}\right)^{2/5}, \text{ for } 0 \le x \le 100.$$

Calculate the median lifetime of a 50-year-old.

$$\text{GDM} \Rightarrow T_{50} \sim \text{GDM with } S_{50}(t) = \left(1 - \frac{t}{50}\right)^{2/5}, \ 0 \le t \le 50$$

Let $m =$ median of T_{50}

$$S_{50}(m) = \left(1 - \frac{m}{50}\right)^{2/5} = \frac{1}{2}$$

$$\Rightarrow m = 50\left(1 - \left(\frac{1}{2}\right)^{5/2}\right)$$

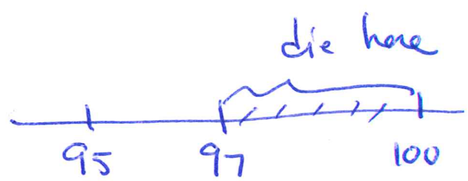
$$= \cancel{12.10709} \quad 41.16117$$

Question No. 7:

Suppose you are given the following extract from a life table:

x	ℓ_x	
94	15,000	
95	12,500	
96	8,750	
97	4,375	2845
98	1,530	
99	380	1150
100	40	340
101	0	

Calculate ${}_{2|3}q_{95}$.



$$ {}_{2|3}q_{95} = \frac{d_{97} + d_{98} + d_{99}}{\ell_{95}} $$

$$ = \frac{2845 + 1150 + 340}{12500} $$

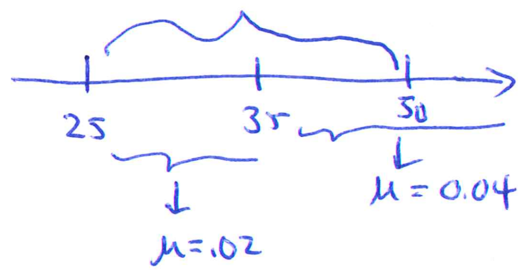
$$ = \underline{\underline{0.3468}} $$

Question No. 8:

Suppose you are given:

$$\mu_x = \begin{cases} 0.02, & 0 < x < 35 \\ 0.04, & x \geq 35 \end{cases}$$

Calculate $\mathring{e}_{25:\overline{25}|}$.



$$\mathring{e}_{25:\overline{25}|} = \mathring{e}_{25:\overline{10}|} + {}_{10}p_{25} * \mathring{e}_{35:\overline{15}|}$$

$$= \int_0^{10} e^{-.02t}\,dt + e^{-.02(10)} * \int_0^{15} e^{-.04t}\,dt$$

$$= \frac{1}{.02}\left(1 - e^{\overset{.2}{-.02(10)}}\right) + e^{-.2} \times \frac{1}{.04}\left(1 - e^{\overset{.6}{-.04(15)}}\right)$$

$$= \frac{1}{.02}\left(1 - e^{-.2}\right) + e^{-.2} \times \frac{1}{.04}\left(1 - e^{-.6}\right)$$

$$= \underline{\underline{18.29851}}$$

Question No. 9:

For a fixed age x, you are given the following probabilities:

- $p_x = 0.95$
- ${}_3q_x = 0.24$

Calculate the probability that $(x+1)$ will not survive the following two years.

$${}_3p_x = p_x * {}_2p_{x+1}$$

$$\Rightarrow {}_2q_{x+1} = 1 - \frac{{}_3p_x}{p_x} = 1 - \frac{0.76}{0.95}$$

$$= \underline{\underline{0.20}}$$

Question No. 10:

The force of mortality for a substandard life (x) is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + a,$$

for some constant $a > 0$, where μ_{x+t} is the force of mortality of a standard life (x).

You are given:

- The probability that a standard life (x) survives the next 20 years is 0.50.
- The probability that a substandard life (x) survives the next 20 years is 0.25.

Calculate the value of the constant a.

Standard life: $\quad {}_{20}p_x = e^{-\int_0^{20} \mu_{x+s}\,ds} = 0.50$

substandard life: $\quad {}_{20}p^s_x = e^{-\int_0^{20} \mu^s_{x+s}\,ds}$

$$= e^{-\int_0^{20} (\mu_{x+s} + a)\,ds}$$

$$= \underbrace{e^{-\int_0^{20} \mu_{x+s}\,ds}}_{0.50}\; e^{-20a} = 0.25$$

Solving for a, we get $\quad e^{-20a} = 0.50$

$$20a = -\log(0.50) = \log(2)$$

$$a = \frac{1}{20}\log(2) = \underline{\underline{0.0346573 6}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK