MATH 3630
Actuarial Mathematics I
Class Test 1 - 3:35-4:50 PM
Wednesday, 30 September 2015
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: Emil

Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

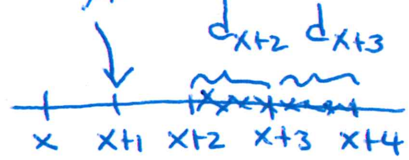
Question No. 1:

For a life (x), you are given $\ell_x = 20,000$ and the following extract from a life table:

k	d_{x+k}
0	250
1	500
2	700
3	1000
4	1500

Calculate ${}_{1|2}q_{x+1}$ and interpret this probability.

$$\ell_{x+1} = \ell_x - d_x = 20{,}000 - 250 = 19{,}750$$



$$ {}_{1|2}q_{x+1} = \frac{d_{x+2} + d_{x+3}}{\ell_{x+1}} = \frac{700 + \overset{1000}{\cancel{500}}}{19750} = \overset{0.08607595}{\cancel{0.06075949}} $$

This is the probability a life $(x+1)$ survives for one year, but dies the following two years, or a life $(x+1)$ dies between ages $x+2$ and $x+4$.

Question No. 2:

Suppose the force of mortality is:

$$\mu_x = c + e^x, \quad \text{for } x \geq 0 \text{ and } c > 0.$$

You are given: $p_0 = 0.1623$.

Calculate the value of c.

First derive ${}_tp_x = \exp\left[-\int_0^t \mu_{x+s}\,ds\right] = \exp\left[-\int_0^t (c + e^x e^s)\,ds\right]$

$$= \exp\left(-ct - e^x(e^t - 1)\right)$$

Thus

$$p_0 = \exp\left(-c - e^0(e-1)\right) = \exp(-c - e + 1) = 0.1623$$

$t=1$
$x=0$

Solving for c, we get

$$c = 1 - e - \log(0.1623)$$

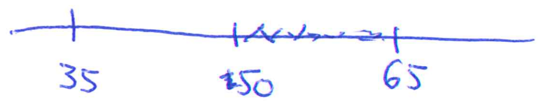
$$= 0.100027 \approx \underline{\underline{0.10}}$$

Question No. 3:

You are given:

- The probability that (35) survives to reach age 50 is 0.83.
- The probability that (35) dies between the ages of 50 and 65 is 0.15.
- $\ell_{65} = 6800$

Calculate ℓ_{35}.



$${}_{15}p_{35} = 0.83$$

$${}_{15|10}q_{35} = {}_{15}p_{35}\ {}_{15}q_{50} = 0.15 \Rightarrow {}_{15}q_{50} = \frac{0.15}{0.83} = \frac{15}{83}$$

$$\Rightarrow {}_{15}p_{50} = 1 - \frac{15}{83} = \frac{68}{83}$$

Thus

$$\frac{\ell_{65}}{\ell_{50}} = \frac{68}{83} \Rightarrow \ell_{50} = \overset{100}{\cancel{6800}} \times \frac{83}{\cancel{68}} = 8,300$$

$${}_{15}p_{35} = \frac{\ell_{50}}{\ell_{35}} = 0.83 \Rightarrow \ell_{35} = \frac{\ell_{50}}{0.83} = \frac{8300}{0.83}$$

$$= \underline{\underline{10,000}}$$

Question No. 4:

You are given:

$$q_{65+k} = 0.02, \quad \text{for } k = 0, 1, 2, \ldots$$

Calculate e_{65}, the curtate expectation of life for a person age 65.

$$e_{65} = \sum_{K=1}^{\infty} {}_{k}p_{65}$$

$$= \sum_{K=1}^{\infty} p_{65} \cdot p_{66} \cdots p_{65+k-1}$$

$$= \sum_{K=1}^{\infty} (.98)^k = \frac{.98}{1-0.98} = \underline{\underline{49}}$$

Question No. 5:

Suppose the survival function for a newborn is given by

$$S_0(x) = e^{-0.002x^2}, \quad \text{for } x \geq 0.$$

Calculate μ_{40}.

$$\mu_x = -\frac{d}{dx} \log S_0(x) = \frac{d}{dx}(.002x^2) = .004x$$

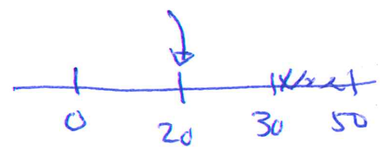
$$\mu_{40} = .004(40) = \underline{\underline{0.16}}$$

Question No. 6:

Mortality follows the Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{100}\right)^{2/3}, \text{ for } 0 \le x \le 100.$$

Calculate the probability that life (20) will die between ages 30 and 50.



$$ {}_{10|20}q_{20} = \frac{S_0(30) - S_0(50)}{S_0(20)}$$

$$= \frac{\left(\frac{70}{100}\right)^{2/3} - \left(\frac{50}{100}\right)^{2/3}}{\left(\frac{80}{100}\right)^{2/3}} \approx \underline{\underline{0.183822}}$$

This is equivalent to $S_{20}(10) - S_{20}(30) = \left(\frac{70}{80}\right)^{2/3} - \left(\frac{50}{80}\right)^{2/3}$

where $S_{20}(t) = \left(1 - t/80\right)^{2/3}$ $= \underline{\underline{0.183822}}$

Since $T_{20} \sim GDM(\alpha = 2/3, \omega = 80)$

Question No. 7:

Suppose you are given the following extract from a life table:

x	ℓ_x
94	15,000
95	12,500
96	8,750
97	4,375
98	1,530
99	380
100	40
101	0

Calculate $e_{97:\overline{3}|}$.

$$e_{97:\overline{3}|} = \sum_{K=1}^{3} {}_K p_{97} = \frac{l_{98} + l_{99} + l_{100}}{l_{97}}$$

$$= \frac{1530 + 380 + 40}{4375}$$

$$= \underline{\underline{0.4457143}}$$

Question No. 8:

The force of mortality for a substandard life (x) is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + c,$$

for some constant $c > 0$, where μ_{x+t} is the force of mortality of a standard life (x). You are given:

- The probability that a standard life (x) survives the next 5 years is 0.75.
- The probability that a substandard life (x) survives the next 5 years is 0.40.

Calculate the value of c.

For the standard life, ${}_5p_x = e^{-\int_0^5 \mu_{x+s} ds} = 0.75$

For the substandard life,

$$ {}_5p_x^s = e^{-\int_0^5 (\mu_{x+s}+c) ds} $$

$$ = \underbrace{e^{-\int_0^5 \mu_{x+s} ds}}_{{}_5p_x} \cdot e^{-5c} = 0.75 e^{-5c} = 0.40 $$

Thus, we have

$$ e^{-5c} = \frac{0.40}{0.75} = \frac{40}{75} \Rightarrow c = -\frac{1}{5}\log\left(\frac{40}{75}\right) $$

$$ = \underline{\underline{0.1257217}} $$

Question No. 9:

For a fixed age x, you are given the following probabilities:

- $p_x = 0.98$
- $p_{x+1} = 0.97$
- ${}_3p_{x+1} = 0.866$
- $q_{x+3} = 0.06$

Calculate ${}_3p_x$.

This is similar to Exercise 2.6 of book!

$${}_3p_x = p_x \cdot p_{x+1} \cdot p_{x+2}$$

$${}_3p_{x+1} = p_{x+1} \cdot p_{x+2} \cdot p_{x+3} \implies p_{x+1} \cdot p_{x+2} = \frac{{}_3p_{x+1}}{p_{x+3}} = \frac{0.866}{.94}$$

Thus,

$${}_3p_x = (0.98)(0.866/0.94)$$

$$= \underline{\underline{0.9028511}}$$

Question No. 10:

In a population consisting of 60% males and 40% females, you are given:

- Mortality for females has a constant force of μ.
- Mortality for males also has a constant force of 3μ, three times that of females.
- Out of the survivors at the end of 10 years, the proportion of males is 50%.

Calculate the probability a female, of any age, survives a year.

The proportion of males at end of 10 years is

$$\frac{.60e^{-30\mu}}{.60e^{-30\mu}+.40e^{-10\mu}} = \frac{.60e^{-20\mu}}{.60e^{-20\mu}+.40} = 50\%$$

Solving for $e^{-20\mu}$, we get

$$\cancel{.50}(.60)e^{-20\mu} = \cancel{.50}(.40)$$

$$e^{-20\mu} = \frac{.40}{.60} = \frac{2}{3}$$

$$\Rightarrow \quad e^{-\mu} = \left(\frac{2}{3}\right)^{1/20}$$

The probability a female survives a year is

$$e^{-\mu} = \left(\frac{2}{3}\right)^{1/20} = 0.9799309 \approx \underline{\underline{0.98}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK