NAME (Print neatly!): _____

MATH 3160 (Roby) **Practice Quiz #1 Solutions** January 2022

For each problem below, please explain your solution and give your final answer in two forms: (a) Expressed in terms of numbers and symbols with combinatorial significance for the problem (e.g., $2\binom{13}{5} + (n!)^2$) and (b) the actual numerical answer (if you have time). For example, in five-card poker, the number of two pair hands is given by (a) $13 \cdot \binom{4}{2} \cdot 12 \cdot \binom{4}{2} \cdot \frac{1}{2} \cdot 11 \cdot \binom{4}{1}$ and (b) 123552.

1. What is the coefficient of x^4y^3 in $(2x^2 - 3y)^5$? Using the binomial theorem, we see that we need the coefficient $\binom{5}{2}2^2(-3)^3 = -1080$.

2. There are four Vulcans and six Klingons. Each Vulcan (monogamously) marries one of the Klingons. In how many ways can this be done? There first Vulcan has six choices, the next has five, and so on, meaning $6 \cdot 5 \cdot 4 \cdot 3 = 360$ total possibilities.

3. In how many ways can the letters of the word *BOOKKEEPER* be arranged if the three *E*'s cannot be listed consecutively? This is permutation with repetitions. There are 10! ways to permute the letters, but ones which differ in just an arrangement of the 3 E's will look the same and are counted 3! times. Similarly we must divide by the 2! ways to arrange the K's and (separately) the O's. This gives the first term in the expression below.

   Next we subtract the number of such arrangements where the three E's are consecutive, meaning we can treat them as a single "block" or symbol to be rearranged with the other 7 symbols. The same reasoning gives the term we subtract off.

   Finally, we get $\dfrac{10!}{3!2!2!} - \dfrac{8!}{2!2!} = 151,200 - 10080 = 141120$.

NB: During an in-person quiz, it will usually be enough enough to write the symbolic answer, especially if the numbers are large.