Midterm #2 REWRITE (DUE 16 November 2017)

- **You may rewrite any problem or sub-problem on which you lost a total of 2 or more points.**
- **Please rewrite the entire answer to each part of a problem you'd like considered for extra points, even if it means copying things you already did correctly. Otherwise I have to check back and forth, which takes much more time. If you already got a part of a problem completely correct, just say something like "(a) received full credit".**
- **Double check your answers (with friends or however). Often you can see whether your answer makes sense just by checking it against the original problem.**
- **You may discuss problems with your classmates or others, but please indicate your source if you get significant help. (It won't affect your score.)**
- **Expect the grading of rewrites to be tougher than the in-class exam. You have ample time to check your answers for clarity and completeness. Your rewrites should be perfect.**
- **You can receive up to half the points you missed on any problem. Ideally, I will look at your rewrite, note that it's correct and complete, and give you half of all the points you missed.**

Extra Credit Problem (5pts): After you finish rewriting the exam questions, write a few paragraphs on the back page (page 10), detailing what specific course content you found most confusing, and anything that helped you straighten it out as you did the rewrites. Please add any other comments about the flipped format that might be relevent to your learning. (Goals here include (1) for me to learn how to better teach the material next time; and (2) for you to reflect on the course content.)

1. Suppose $A = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & 6 \\ 4 & 9 & -12 & 9 & 3 & 12 \\ -2 & 3 & 6 & 3 & 3 & -6 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ 0 & 3 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(a) Compute $\operatorname{rank} A$; explain!

(b) Compute $\dim \operatorname{Nul} A$; explain!

(c) Find a basis for $\operatorname{Col} A$; explain!

(d) Find a basis for $\operatorname{Row} A$; explain!

2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & 1 \end{bmatrix}$.

(a) Compute $\det A$. (HINT: Avoid cofactor expansion!)

(b) Compute $\det A^3$.

(c) Can you use Cramer's rule to calculate a value of x_2 which solves the matrix equation $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 0 \\ 0 \end{bmatrix}$? Explain why or why not. (You do not need to actually compute x_2.)

(d) Find the volume of the *4-dimensional* parallelpiped given by the following vectors.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}, \text{ and } \begin{bmatrix} -1 \\ -2 \\ -3 \\ 1 \end{bmatrix}.$$

3. Let $A = \begin{bmatrix} 0 & 3 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 3 & 0 & 0 & -1 \end{bmatrix}$.

(a) Is A invertible? Is B invertible? Explain!

(b) Is A diagonalizable? What would D be? Explain!

(c) Is B diagonalizable? What would D be? Explain!

4. For each statement below circle whether it is **True** or **False** and give **reasons** to support your answer. To show something is false, it is usually best to give a **specific simple numeric counterexample**. Extra credit for "salvaging" false statements (when possible) to make them correct.

   (a) **True or False:** If 0 is an eigenvalue of a square matrix A, then A is not invertible.

   (b) **True or False:** If λ and μ are eigenvalues of the same $n \times n$ matrix M, then $\lambda + \mu$ is an eigenvalue of M.

   (c) **True or False:** If $A \in \mathbb{R}^{n \times n}$ satisfies $A^4 = 0$, then $\det A = 0$.

(d) **True or False:** If two matrices A and B have the same row-reduced echelon form, then $\text{Row}\, A = \text{Row}\, B$.

(e) **True or False:** If a finite set S of nonzero vectors spans a vector space V, then S is a basis for V.

(f) **True or False:** If two matrices A and B are *similar* (i.e., there exists an invertible matrix P such that $A = PBP^{-1}$), then $\det A = \det B$.

5. Determine whether the following subsets S of the given vector space V are **subspaces** of V. Justify your answers!

(a) $V = \mathbb{P}_4$ and $S = \{\mathbf{p}(t) = a + at^2 : a \in \mathbb{R}\}$.

(b) $V = \mathbb{P}_3$ and $S = \{a(1-t^2) + b(1-t) + t^3 : a, b \in \mathbb{R}\}$.

(c) $V = \mathbb{R}^3$ and $S = \left\{ \begin{bmatrix} a \\ b \\ a^2 \end{bmatrix} : a, b \in \mathbb{R} \right\}$.

(d) $V = \mathbb{R}^4$, and $S =$ the set of all eigenvectors corresponding to the eigenvalue 2 for the matrix $\begin{bmatrix} -2 & 3 & -2 & 1 \\ 0 & 2 & -4 & 3 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

6. The set $\mathcal{B} = \{1 - t^2, 1 + 2t - t^2, 2 - 3t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 8 - 2t^2$ relative to $\mathcal{B}$.

7. An engineer solves a nonhomogenous system of twelve linear equations in fourteen unknowns, where three of the unknowns are free variables. Can she be certain when the right sides of the equations are changed, that the new nonhomogenous system will also have a solution?

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