Video Lecture E1: Systems of Linear Equations

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Outline & Objectives

- Recognize systems of linear equations.
- Recall using *elimination* simplify or solve a system of linear equations.
- Analyze possible solution sets for a linear system;
- Learn the notions of *consistency* (solution *exists*) and *uniqueness* (*at most one* solution)

What does linear mean?

A *linear equation* (in n variables) can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b,$$

where $a_i, b \in \mathbb{R}$. (Combine like terms if necc.) A *linear system* is one or more linear eqns. considered simultaneously.

Q: Which of the following are linear systems?

$\begin{cases} 2x + 3y = 4 \\ 4x + 7y = 6 \end{cases}$? $\begin{cases} 2x + 3y = 4y \\ 4x = 6 \end{cases}$?

$\begin{cases} 2x + 3y - 8z = 4y + 9x \\ 4x + 7y = 6xy \end{cases}$? $\begin{cases} (2 + \sqrt{7})x + 3y = \pi \\ 4x + 7y = 6 \end{cases}$?

$\begin{cases} 2x_1 + 3x_2 - 5x_3 = 4 \\ 4x_1 - 7x_3 = 4 \\ x_2 - 4x_3 = 4 \end{cases}$

How to solve a linear system?

EG: $\begin{cases} 2x + 3y = 4 \\ 4x + 7y = 6 \end{cases}$

What can happen?

Theorem

A system of linear equations can have:

1. *exactly one (a unique) solution (consistent);*
2. *infinitely many solutions (consistent); or*
3. *no solution (inconsistent).*

$$\begin{cases} x_1 + x_2 = 8 \\ x_1 - x_2 = 0 \end{cases} \qquad \begin{cases} 2x_1 - x_2 = -3 \\ 4x_1 - 2x_2 = 5 \end{cases} \qquad \begin{cases} x_1 + x_2 = 3 \\ 2x_1 + 2x_2 = 6 \end{cases}$$