Algebraic Combinatorics HW #2 DUE: 17 September 2013

From the text

Ch. 1: #3, 6, 7, and 12a. Please write careful solutions to these problems.

Extra Credit: Ch. 1, #1.

Other Exercises

A. Here's an alternate approach to getting the eigenvalues of the cube graph Q_n.

   (a) Let C_n denote the adjacency matrix of Q_n. Compute (e.g., via Sage) C_n for the first few values of n. This should lead you to a conjecture about how to represent C_n as a block matrix in terms of C_{n-1}.

   (b) Compute the eigenvalues and eigenvectors of C_n for $n = 1$ and $n = 2$.

   (c) Let $\{v_1, \ldots v_{2^n}\}$ denote a complete set of eigenvectors for C_n. Show how to use these to construct a complete set of eigenvectors $\{w_1, \ldots\}$ for C_{n+1}. What are the corresponding eigenvalues for C_{n+1} in terms of C_n?

   (d) Now use induction to prove that

$$\operatorname{spec} C_n = \left\{ [n]^{\binom{n}{0}}, [n-2]^{\binom{n}{1}}, \ldots, [n-2k]^{\binom{n}{k}}, \ldots [-n]^{\binom{n}{n}} \right\} .$$

B. Suppose that the adjacency matrix A of a graph G satisfies $A^2 = A$. What can you say about the structure of G?

C. Work through the Sage tutorial

   `http://www.sagemath.org/doc/tutorial/tour_linalg.html`.

   Write down any questions you have after going through it.

D. Search for videos on "Galton Board" and watch a couple. Write down the URL's and let me know if you find one you find particularly good. What is this related to on this problem set?