Algebraic Combinatorics HW #1 DUE: 5 September 2013

From the text

Ch. 1, #2, 4, 9, and 10. Please write careful solutions to these problems.

Sage Exercises

You can do the Sage exercises either (1) locally on your own hardware; (2) in the Sage Cloud `cloud.sagemath.org`, where you create a free account; or (3) at the Sage Notebook Server at `www.sagenb.org`, for which you can create an account or use an existing Google, Yahoo, etc. account. The last option is slowly being deprecated, e.g., it's currently running a much older version, Sage 5.4.

For these exercises, just show me the fruits of your labors; I don't need you to hand in printouts of Sage worksheets.

1. (Optional, but recommended). Install some version of Sage 5.11 on your laptop, desktop, or phone; a binary should be fine, there's probably no need to build from source. Let me know if you have any trouble. For more info see `http://www.sagemath.org/doc/installation/quick-guide.html`.

2. Work through the Sage tutorial `http://sagemath.org/doc/a_tour_of_sage`. Make a note of any math terms you don't understand; feel free to look them up (within Sage or on the web) and/or ask me about them in class.

3. Get sage to produce the quadratic formula (solutions to the equation $ax^2+bx+c=0$).

4. Draw a (non-complete) connected graph with four or five vertices and at least 5 edges. Compute the first ten powers of the matrix (using Sage). Now find a entry in one of these matrices, say A^k where $k \geq 4$ which has a value in the range 7–14. Explain the combinatorial significance in terms of walks on the graph of the entry you pick, and use this to verify the value computed by Sage.

5. Get Sage to show you pictures of the following graphs: (a) the complete graphs K_p for $p \in [2,7]$; (b) the complete bipartite graphs $K_{m,n}$ for $m,n \in [2,4]$; (c) the complete multipartite graphs $K(n,p)$ for $n,p \in [2,4]$; (d) the Petersen graph.

6. Compute the spectrum (multiset of eigenvalues) for each of the above examples. Can you make any conjectures?

7. Compute the eigenvectors for each example above. What patterns do you notice? Does this help you prove any of your conjectures?

8. Let v and w be adjacent vertices in the Petersen graph. How many walks of length 12 are there from v to w? Does this depend on our choice of v and w? Explain!

Linear Algebra Review Exercises

It may be beneficial for us to do some review of linear algebra in class. Please let me know if you have any questions about any of these. Feel free to look things up in a text on linear algebra or on the web.

1. If the eigenvalues of a matrix M are $\lambda_1, \lambda_2, \ldots, \lambda_p$, what are the eigenvalues of M^2? Of M^{-1}? Of M^k for any integer k? (Hint: use the definition.)

2. If the eigenvalues of a matrix M are $\lambda_1, \lambda_2, \ldots, \lambda_p$, what are the eigenvalues of the matrix $M + cI$, where c is a fixed constant and I is the identity matrix?

3. If the eigenvalues of a matrix M are $\lambda_1, \lambda_2, \ldots, \lambda_p$, what are the eigenvalues of the block matrix $\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}$? How about of $\begin{bmatrix} 0 & M \\ M & 0 \end{bmatrix}$? You might want to try some examples in Sage first...

4. Look up the *Kronecker Product* $A \otimes B$ of two matrices A and B. How do the eigenvalues of $A \otimes B$ compare with those of A and B? [No formal proof necessary.]