Math 2710 (Roby) **Practice Midterm #1** Spring 2013

1. Use a truth table to check whether the statement $(P \text{ AND } Q) \implies R$ is equivalent to the statement $P \implies (Q \implies R)$.

2. **PODASIP:** For all integers $a, b \in \mathbb{Z}$, $a \mid b \implies a \leq b$.

3. **PODASIP:** For any sets S and T: $(S \cap T = \emptyset)$ AND $(S \cup T = T) \implies S = \emptyset$.

4. For each linear diophantine equation below, do the following:
   (a) Determine whether it has a solution in $\mathbb{Z}^2$.
   (b) Find one solution.
   (c) Describe the set of all solutions;
   (d) Describe all solutions in *positive* integers.

$$18x + 5y = 48 \tag{1}$$
$$14x + 35y = 93 \tag{2}$$

5. Show that there are infinitely many primes in $\mathbb{Z}$ of the form $4k + 3$.

6. Describe the chain of reasoning that takes one from basic properties of $\mathbb{Z}$ to the Fundamental Theorem of Arithmetic (aka, Unique Factorization Theorem).

7. **True/False & Explain:** For each statement below, state whether it is true or false and give a convincing reason.
   (a) $\forall x, y \in \mathbb{Q}\,,\ \exists z \in \mathbb{Q}$ s.t. $x < z < y$.
   (b) $\exists z \in \mathbb{Q}$ s.t. $\forall x, y \in \mathbb{Q}\,,\ x < z < y$
   (c) $\exists y \in \mathbb{Z}$ s.t. $x + y = x\ \forall x \in \mathbb{Z}$.
   (d) $\exists y \in \mathbb{Z}$ s.t. $\forall x \in \mathbb{Z}\,,\ x + y = 0$.

8. Explain what is wrong with the following proof attempt:

   We want to show that $a \mid b$ AND $b \mid c \implies a \mid c$. *Suppose by way of contradiction that the statement was false, so* $a|b$ *and* $b|c$, *but* $a \nmid b$. *But we see immediately that* $3 \mid 6$ *and* $6 \mid 12$, *while* $3 \mid 12$, ***contradiction***. *Therefore, the original statement must be true.* ∎