Math 227Q/Roby **Practice Final** December 2005

SHOW ALL YOUR WORK! Make sure you give reasons to support your answers. If you have any questions, do not hesitate to ask! No calculators are to be used, but you may bring one $4'' \times 6''$ notecard to class with any notes you like.

1. Find a QR factorization for the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}$, by first applying Gram-Schmidt to the columns of A.

2. Define $T : \mathbb{P}_2 \to R^3$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(-1) \\ \mathbf{p}(0) \\ \mathbf{p}(1) \end{bmatrix}$.

   (a) Find the image under T of $\mathbf{p}(t) = 5 + 3t$.

   (b) Show that T is a linear transformation.

   (c) Find the matrix for T relative to the basis $\{1, t, t^2\}$ for $\mathbb{P}_2$ and the standard basis for $\mathbb{R}^3$.

   (d) Is T one to one? Is T onto? Explain!

3. Find the characteristic polynomial and the eigenvalues of the matrix $\begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}$.

4. Show that the determinant of $T = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ is $\det T = (b - a)(c - a)(c - b)$.

5. Prove or Disprove and Salvage if possible:

   (a) If $A = QR$, where Q has orthonormal columns, then $R = Q^T A$.

   (b) If $S = \{u_1, \ldots, u_p\}$ is an orthogonal set of vectors in $\mathbb{R}^n$, then S is linearly independent.

   (c) Each eigenvector of a square matrix A is also an eigenvector of A^2.

   (d) There exists a 2×2 matrix that has no eigenvectors in $\mathbb{R}^2$.

   (e) If A is row equivalent to the identity matrix I, then A is diagonalizable.

6. Decide whether each statement below is True of False. Justify your answer.

   (a) If $\mathbf{y}$ is in a subspace W, then the orthogonal projection of $\mathbf{y}$ onto W is $\mathbf{y}$ itself.

   (b) For an $m \times n$ matrix A, vectors in $\operatorname{Nul} A$ are orthogonal to vectors in $\operatorname{Row} A$.

   (c) The matrices A and A^T have the same eigenvalues, counting multiplicities.

   (d) A nonzero vector can correspond to two different eigenvalues of A.

   (e) The sum of two eigenvectors of a square matrix A is also an eigenvector of A.

7. If a $n \times n$ matrix A satifies $A^2 = A$, what can you say about the determinant of A?

8. Lay #4.6.4 (p. 269).

9. What would you have to know about the solution set of a homogenous system of 18 linear equations in 20 variables in order to know that every associated nonhomogeneous equation has a solution? Discuss!

10. Go back over your old homework and quizzes to review and make sure you understand any problem on which you lost points.