Math 2210Q (Roby) **Practice Quiz #1 Solutions** 29 January 2014

PLEASE WRITE YOUR NAME AT THE BOTTOM OF THE BACK OF THIS SHEET, NOT ON THE FRONT.

1. Mark each of the following as **True** or **False**. You may give reasoning to support your answer, which may give you partial credit. **To show a statement is false, a specific numerical counterexample is generally best!**

   (a) The vector $2\mathbf{v}_1 + \sqrt{5}\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v_3}$.
   True. It equals $2\mathbf{v_1} + 0 \cdot \mathbf{v}_2 + \sqrt{5}\mathbf{v_3}$

   (b) Asking whether the linear system corresponding to an augmented matrix $[\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{b}]$ has a solution amounts to asking whether $\mathbf{b}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$. True, by writing out the definitions of span and noticing that the single vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}$ translate into the corresponding system of simultaneous equations.

   (c) Whenever a system has free variables, the solution set contains more than one solution.
   False. An inconsisent system can have free variables. For example, if we consider the matrix in #3 below to be the augmented matrix of a system, then x_3 is a free variable, but the bottom row indicates that $0 = 1$, so the system is inconsistent and there are no solutions.

2. Find the general solution of the system whose augmented matrix is as follows:

$$\begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{pmatrix}$$

   Row reduce the system to get:

$$\begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & 0 & -5 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -5 & 4 \\ 0 & 1 & -6 & 5 \end{pmatrix}$$

   Hence the general solution is

$$\begin{aligned} x_1 &= 5x_3 + 4 \\ x_2 &= 6x_3 + 5 \\ x_3 &\text{ is free.} \end{aligned}$$

3. Is the following matrix in reduced echelon form, echelon form, or neither? Explain!

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   This matrix is in reduced echelon form, and the pivots are indicated in red above.