Math 213/Roby **Practice Final Exam** 27 November 2007

1. Go over all the midterm, practice midterm, and midterm rewrite problems. Make sure you know how to do all of these, and understand any places where you didn't get full marks.

2. Go over all the quiz problems. Make sure you know how to do all of these, and understand any places where you didn't get full marks.

3. Go over your old homework problems. Make sure you know how to do all of these, and understand any places where you didn't get full marks.

4. Prove or Disprove and Salvage if Possible:

   (a) If a sequence $\langle x \rangle$ of real numbers is unbounded, then it has a limit.

   (b) If a sequence $\langle x \rangle$ of real numbers is not monotone then it has no limit.

   (c) If a sequence $\langle x \rangle$ of real numbers converges, then there exists $n \in \mathbb{N}$ such that $|x_{n+1} - x_n| < 1/2^n$.

   (d) If $a_n \to 0$ and $b_n \to 0$, then $\sum a_n b_n$ converges.

5. Prove that $c^{1/n} \to 1$ whenever c is a positive real number.

6. A runaway train is hurtling toward a brick wall at the speed of 100 miles per hour. When it is two miles from the wall, a fly begins to fly repeatedly between the trains and the wall at the speed of 200 miles per hour. Determine how far the fly travels before it is smashed.

7. Suppose that $x_n \to 0$ and that $|y_n| \leq 1$ for $n \in \mathbb{N}$. Find the flaw in the following computation for $\lim(x_n y_n)$, and give a valid proof that $\lim(x_n y_n) = 0$:

$$\lim(x_n y_n) = \lim(x_n) \lim(y_n) = 0 \cdot \lim(y_n) = 0\,.$$

8. (Brahmagupta, 7th century CE) When eggs in a basket are removed 2,3,4,5,6 at a time there remain, respectively, 1,2,3,4,5 eggs. When they are taken out 7 at a time, none are left over. Find the smallest number of eggs that could have been contained in the basket.

9. Determine whether $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots$ converges.

10. Prove the following statement in two ways: If $\langle a \rangle$ is a sequence whose terms alternate in sign, converge to 0, and satisfy $|a_{k+1}| \leq |a_k|$ for all k, then the series $\sum_{k=0}^{\infty} a_k$ converges.

    (a) Show that the partial sums form a Cauchy sequence

    (b) Use Proposition 13.18 and the Squeeze Theorem (Thm. 14.6)