Poker Hands

Christopher Hayes

Poker Hands

- The "normal" playing card deck of 52 cards is called the *French* deck.
- The French deck actually came from Egypt in the 1300's and was already present in the Middle East long before then.
- 13 cards per *suit*, 4 suits:
  - Suits are **Spades, Clubs,** Hearts, and Diamonds.
  - *Ranks*: Ace, 2 to 10, Jack, Queen, King
- There are 5 cards in one *hand*.

General Formulas (Pick *k* from *n*)

	Order Matters	Order Doesn't Matter
Replacement	n^k	$\binom{n+k-1}{k} = \frac{(n+k-1)!}{k!\,(n-1)!}$
No Replacement	${}_n\mathrm{P}_k = \frac{n!}{(n-k)!}$ (Permutations)	${}_n\mathrm{C}_k = \frac{n!}{k!(n-k)!} = \binom{n}{k}$ (Combinations)

Total Possible Hands

- Order matters, no replacement.

- Choose 5 from 52:
  - $\frac{52!}{5!(52-5)!} = 2{,}598{,}960$

- There are over 2.5 million possible hands.

Royal Flush

- Ace-King-Queen-Jack-Ten, all of which are the *same* suit.
- There are clearly only 4 of these.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Total Possible Hands	2,598, 960

Straight Flush

- A “run” of 5 cards, all of which are the same suit.
- Within one suit, how many possible “starting” cards are there?
- Ace, 2, 3, … , 10 are possible.
  – Can start with Ace or end with Ace.
- Four suits, so 4 times 10 is 40.
  – Subtract the 4 Royal Flushes to get 36.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Total Possible Hands	2,598, 960

Four of a Kind

- There are thirteen ranks, so 13 possible four of a kinds.

- However, a four of a kind's *fifth* card can vary! How many possible choices for the fifth card?
- There are 52 – 4 = 48 cards left, so 48 choices.
- 13 times 48 is 624.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Four of a Kind	624
Total Possible Hands	2,598, 960

Full House

- Three of the same rank, plus two that are the same of a different rank.
- Pick one of the 13 ranks.
  - There are 4 choices, we only need 3, so 4 ways.
  - 13 times 4 = 52 ways for the first 3 cards.
- Pick one of the 12 remaining ranks.
  - There are four choices and we need 2, so 6 ways.
  - 12 times 6 is 72 ways for the last 2 cards.
- There are 52 times 72 = 3744 full houses.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Four of a Kind	624
Full House	3744
Total Possible Hands	2,598, 960

Flush

- Five cards from the same suit.
- Within one suit, we simply select 5 from 13.
  - $\frac{13!}{5!8!} = 1287$
- There are four suits, so:
  - 1287 times 4 is 5148 possible flushes.

- Subtract the Royal and Straight Flushes:
  - Final answer is 5148 – 40 = 5108 “common” flushes.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Four of a Kind	624
Full House	3744
Flush	5108
Total Possible Hands	2,598, 960

Straight

- A *straight* is five cards in sequential order, the suit does not matter.
- Recall from the Straight Flush slide that we have 10 starting positions.
- From each starting position, we have 4 choices.
- There are $10 \cdot 4^5 = 10{,}240$ ways.
- Remove Royal and Straight flushes: 10,200.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Four of a Kind	624
Full House	3,744
Flush	5,108
Straight	10,200
Total Possible Hands	2,598, 960

Three of a Kind

- Remember from the Full House slide that there are 52 ways to pick 3 cards of the same rank.
- For the last 2 cards, we have 48 and then 47 choices, so:
  - $\frac{52 \cdot 49 \cdot 48}{2} = 58656$
- Remove the Full Houses to get 54,912 ways.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Four of a Kind	624
Full House	3,744
Flush	5,108
Straight	10,200
Three of a Kind	54,912
Total Possible Hands	2,598, 960

Two Pair

- Similar to the Full House:
- For the first pair:
  - From one of the 13 ranks, pick 2 from 4, so
    - 6 times 13 = 78 ways.
- For the second pair:
  - Pick 2 from 4, for one of the 12 remaining ranks:
    - 6 times 12 = 72 ways.
- For the last card there are 44 (took 4 cards, eliminated 4 cards) possibles, so:
- There are $\frac{78 \cdot 72 \cdot 44}{2} = 123{,}552$ two pairs.

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Four of a Kind	624
Full House	3,744
Flush	5,108
Straight	10,200
Three of a Kind	54,912
Two Pair	123,552
Total Possible Hands	2,598, 960

Pair

- For the pair, we have 78 ways (see Two Pair's slide).

- For the remaining three cards, we choose from 48 then 44 then 40, in 6 possible orders:
  - Total Pairs: $78 \cdot \frac{48 \cdot 44 \cdot 40}{6} = 1{,}098{,}240$ ways

Hands Chart

Type of Hands	Number of Possible Hands
Royal Flush	4
Straight Flush	36
Four of a Kind	624
Full House	3,744
Flush	5,108
Straight	10,200
Three of a Kind	57,408
Two Pair	123,552
Pair	1,098,240
Total Possible Hands	2,598, 960

High Card

- The total number of possible hands was calculated as: 2,598,960
- Subtract all meaningful hands to get all remaining hands (i.e. High Card hands).
- We have 1,302,540 high card hands.

Hands Chart

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Royal Flush	4
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Four of a Kind	624
Full House	3,744
Flush	5,108
Straight	10,200
Three of a Kind	57,408
Two Pair	123,552
Pair	1,098,240
High Card	1,302,540
Total Possible Hands	2,598, 960