Math 1132 Worksheet 11.2 Name: ________________________ Discussion Section: _____

11.2 Series

Divergence Test. If $\lim_{n\to\infty} a_n$ either does not exist or is not 0 then $\sum_{n=1}^{\infty} a_n$ diverges (that means the partial sums do not have a limit).

Geometric Series. A geometric series $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots$, also written as $\sum_{n=1}^{\infty} ar^{n-1}$, is convergent if $|r| < 1$ and its value is $\frac{a}{1-r}$. If $|r| \geq 1$, the geometric series is divergent.

Example: Compute $\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n$ if it converges or indicate why it diverges.

Thinking about the problem:

This series starts off as $1 + 4/11 + (4/11)^2 + (4/11)^3 + \cdots$ and has the form of a geometric series. We will determine a and r and then use the formula for the value of a geometric series if $|r| < 1$.

Doing the problem:

We have $\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n = \sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots$ for $a = 1$ and $r = 4/11$. Since $0 < r < 1$, $\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n$ converges and is equal to $\frac{a}{1-r} = \frac{1}{1-4/11} = \frac{11}{7}$.

Solutions should show all of your work, not just a single final answer.

1. For the following geometric series, determine a and r and then either compute the series or write that the series is divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{4^{n+3}}$.

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{3^{2n+1}}$.

2. Determine all x for which $\displaystyle\sum_{n=0}^{\infty}(2x)^n$ converges, and when it converges determine its value in terms of x.

3. Express the repeating decimal $.9\overline{34} = .934343434\ldots$ with the help of a geometric series and use the formula for a geometric series to write the decimal as a fraction in reduced form.

4. We want to compute $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+3}\right)$ by determining a formula for its partial sums.

   (a) Write the first five terms of $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+3}\right)$.

   (b) Determine a formula for the Nth partial sum $\sum_{n=1}^{N}\left(\frac{1}{n} - \frac{1}{n+3}\right)$.

   (c) Compute the limit of the Nth partial sum in part (b) as $N \to \infty$.

5. T/F (with justification): If $a_n \to 0$ as $n \to \infty$ then the series $\sum_{n=1}^{\infty} a_n$ converges.