Math 1132 Worksheet 11.1 Name: ________________________ Discussion Section: _____

11.1 Sequences

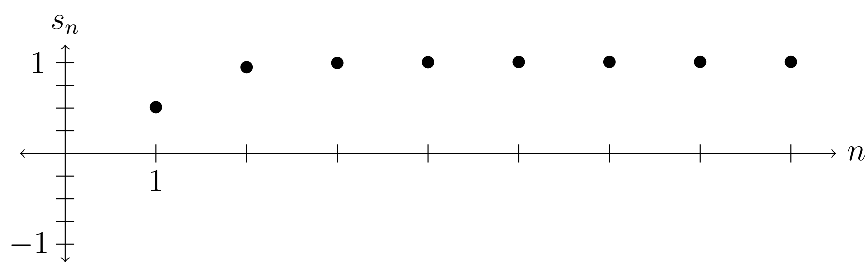
Remember: A *sequence* is a list of numbers $a_1, a_2, a_3, \ldots$. We call it *increasing* if $a_n < a_{n+1}$ for all n, *decreasing* if $a_n > a_{n+1}$ for all n, and *monotonic* if it is either increasing for all n or decreasing for all n. If the terms a_n have a limiting value as $n \to \infty$ then we say that the sequence *converges* (or is *convergent*). Otherwise, the sequence *diverges* (or is *divergent*).

Example: Is the sequence $a_n = \dfrac{n^4}{n^4+1}$ convergent or divergent?

Thinking about the problem:

Let's compute and then plot some values to get a sense of what a_n looks like. See below.

n	1	2	3	4	5	6	7	8
a_n	.5	.94118	.9878	.99611	.9984	.99923	.99958	.99976



The values appear to be tending to 1.

Doing the problem:

Factor out the highest degree part of the numerator and denominator:

$$\frac{n^4}{n^4+1} = \frac{\cancel{n^4} \cdot 1}{\cancel{n^4}(1+1/n^4)} = \frac{1}{1+1/n^4},$$

which tends to 1 as $n \to \infty$. Thus the sequence converges with limit 1.

Solutions should show all of your work, not just a single final answer.

1. For each of the following recurrence relations compute a_n for $n = 1, 2, \ldots, 5$ and then find an explicit formula for a_n in terms of n and determine if the sequence is monotonic.

   (a) $a_{n+1} = 2 - a_n$ where $a_1 = 2$.

n	1	2	3	4	5
a_n					

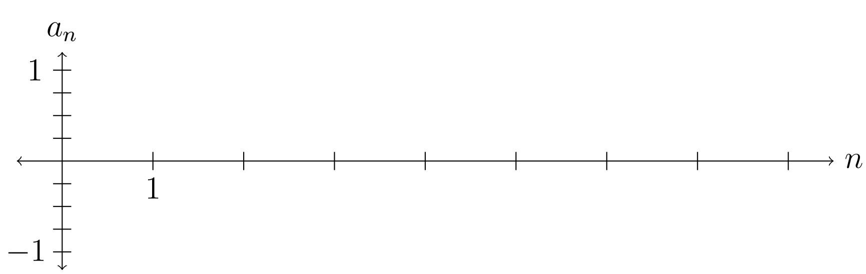
   (b) $a_{n+1} = 3a_n$ where $a_1 = 2$.

n	1	2	3	4	5
a_n					

2. Let $a_n = \dfrac{(-1)^{n-1}}{n}$.

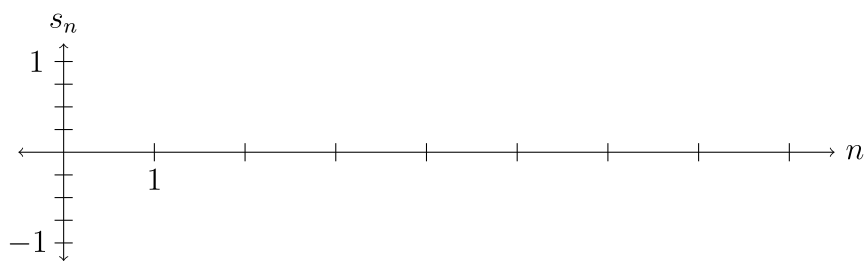
   (a) Compute a_n rounded to 3 digits after the decimal point and plot it against n for $n = 1, 2, \ldots, 8$.

n	1	2	3	4	5	6	7	8
a_n								



(b) Compute $s_n = \sum_{k=1}^{n} a_k$ for $n = 1, 2, \ldots, 8$ rounded to three digits after the decimal point and then plot s_n vs. n for $n = 1, 2, \ldots, 8$.

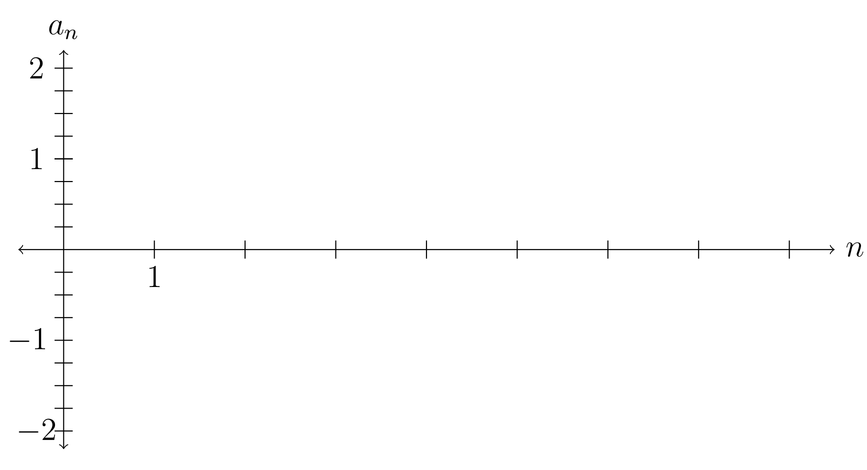
n	1	2	3	4	5	6	7	8
s_n								



3. Let $a_n = \left(1 + \frac{1}{2n}\right)^n$ for $n \geq 1$. We want to determine if this sequence converges, and if it does then show calculations that lead to the limit.

(a) Compute a_n rounded to 3 digits after the decimal point and plot it against n for $n = 1, 2, \ldots, 8$.

n	1	2	3	4	5	6	7	8
a_n								



(b) From the data in (a), what seems to be an approximation to the limit of the terms a_n?

(c) Calculate $\lim_{n\to\infty} a_n$.

(*Hint:* As $n \to \infty$, $\left(1 + \frac{1}{n}\right)^n \to e$. Rewrite $\left(1 + \frac{1}{2n}\right)^n$ as $\left(\left(1 + \frac{1}{2n}\right)^{2n}\right)^{1/2}$ and remember that as $n \to \infty$ also $2n \to \infty$.)

4. Determine the limit of the sequence $a_n = \dfrac{\cos n}{\sqrt{n}}$ or state the limit does not exist. If there is a limit, show calculations that explain how you find the limit.

5. T/F (with justification): Every bounded sequence is convergent.