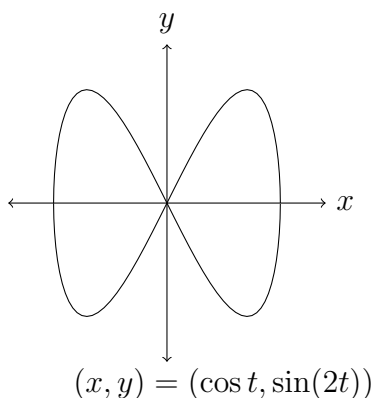

10.2 Calculus with Parametric Curves

Tangents on Parametric Curves. On a parametric curve $(x(t), y(t))$, the slope of a tangent line is $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ at points where $\frac{dx}{dt} \neq 0$.

Example: On the parametric curve $(x, y) = (\cos(t), \sin(2t))$, whose graph is below, determine (a) the slopes of the two tangent lines at the origin and (b) coordinates of the point in the first quadrant where the tangent line has slope -2 .

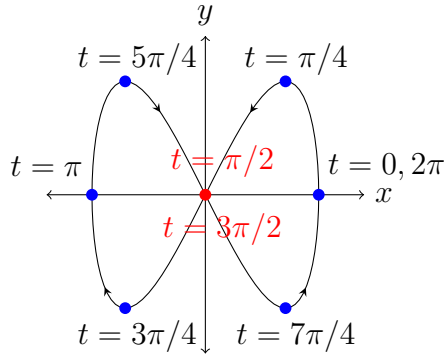


Thinking about the problem:

We will figure out t -values where $(x(t), y(t))$ is the origin and compute $\frac{dy}{dx}$ at such t . To figure out where $\frac{dy}{dx} = -2$ in the first quadrant, rewrite this as $\frac{dy}{dt} = -2\frac{dx}{dt}$, find the t -value where that happens in the first quadrant, and then compute $(x(t), y(t))$ to get coordinates.

Doing the problem:

Below we mark points at t -values that are multiples of $\pi/4$ from 0 to 2π . Starting at $(1, 0)$ where $t = 0$, the curve is traced out through quadrants 1, 3, 2, and 4 (note the arrows) before returning to $(1, 0)$.



(a) The derivative on this curve is $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2 \cos(2t)}{-\sin t} = -2 \frac{\cos(2t)}{\sin t}$. The two times

the curve passes through the origin are at $t = \pi/2$ and $t = 3\pi/2$, and the derivatives at these t -values are

$$\left. \frac{dy}{dx} \right|_{t=\pi/2} = -2 \frac{\cos(2(\pi/2))}{\sin(\pi/2)} = -2 \frac{\cos(\pi)}{\sin(\pi/2)} = -2 \left(\frac{-1}{1} \right) = \boxed{2}$$

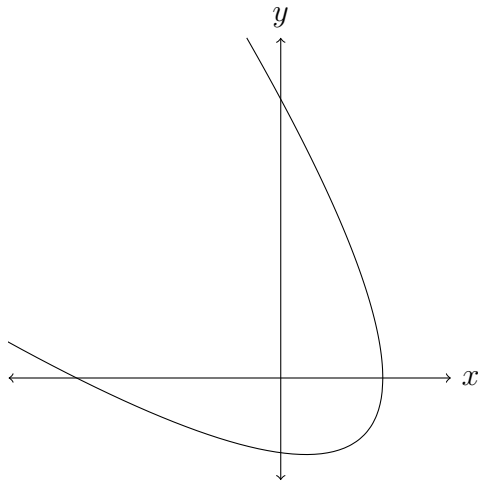
and

$$\left. \frac{dy}{dx} \right|_{t=3\pi/2} = -2 \frac{\cos(2(3\pi/2))}{\sin(3\pi/2)} = -2 \frac{\cos(3\pi)}{\sin(3\pi/2)} = -2 \left(\frac{-1}{-1} \right) = \boxed{-2}.$$

(b) To find where $\frac{dy}{dx} = -2$ in the first quadrant we will solve $-2 \frac{\cos(2t)}{\sin t} = -2$ with $0 \leq t \leq \pi/2$. That means $\cos(2t) = \sin t$, or $1 - 2\sin^2(t) = \sin t$ by the double-angle formula for $\cos(2t)$. By the quadratic formula, $1 - 2a^2 = a$ at $a = 1/2$ and -1 , so we want to solve $\sin t = 1/2$ and $\sin t = -1$. When $0 \leq t \leq \pi/2$ the number $\sin t$ is not negative so we just need to solve $\sin t = 1/2$ and that happens at $t = \pi/6$. Thus the point in quadrant 1 with tangent slope -2 is $(x(\pi/6), y(\pi/6)) = (\cos(\pi/6), \sin(\pi/3)) = \boxed{(\sqrt{3}/2, \sqrt{3}/2)}$.

Solutions should show all of your work, not just a single final answer.

Pictured below is the parametric curve $(x, y) = (3 - t^2, t^2 + 3t)$. It is a rotated parabola.



1. Find some points to let you mark the direction of increasing t -values on the curve.
2. Express dy/dx in terms of the parameter t .
3. Find the slope of the tangent line at the point on the curve where it crosses the positive y -axis.

4. Find coordinates of the point on the curve where the tangent line is horizontal. (First, as a reality check, see which quadrant your answer should be in.)

5. T/F (with justification)

On the parametric curve $(x, y) = (t^2 - 2t, t^3 - 3)$ the graph is increasing at the point where $t = 1/2$.