Important Notice: To prepare for the final exam, one should study the past exams and practice midterms (and homeworks, quizzes, and worksheets), not just this practice final. A topic not being on the practice final does not mean it won't appear on the final.

1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

   (a) If a force of $F(x) = 6x$ pounds is required to stretch a spring x feet beyond its rest length, then 36 ft-lbs of work is done in stretching the spring from its natural length to 6 feet beyond its rest length. False

   (b) The trapezoid rule with $n = 5$ for $\displaystyle\int_0^4 \frac{dx}{2x+1}$ will be an overestimate. True

   (c) $\ln(2.5) = 1.5 - \frac{1}{2}(1.5)^2 + \frac{1}{3}(1.5)^3 - \frac{1}{4}(1.5)^4 + \frac{1}{5}(1.5)^5 - \dots$ False

   (d) The improper integral $\displaystyle\int_1^\infty \frac{x^2}{(x^3+7)^{1/3}}\,dx$ converges.

   (e) The tangent line to the parametric curve $(x, y) = (t - 1/t, 4 + t^2)$ at the point where $t = 1$ has equation $y = x + 5$. True

2. For each multiple choice question, circle the correct answer. There is only one correct choice for each answer.

   (a) A cylindrical tank with a radius of 1 meter and a height of 8 meters is half full. Letting $y = 0$ correspond to the top of the tank and ρ be the density of water, the work required to pump the water out of the tank is

   (a) $\pi\rho g \displaystyle\int_4^8 y\,dy$ (b) $\pi\rho g \displaystyle\int_0^8 y\,dy$ (c) $\pi\rho g \displaystyle\int_0^4 y\,dy$ (d) $16\pi\rho g \displaystyle\int_4^8 y\,dy$

   (b) The Taylor series at $x = 0$ for $\sin x$ is

   (a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{n!}$ (b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$ (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{(2n-1)!}$ (d) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$

   (c) A parametric curve tracing out the circle once clockwise for $0 \le t \le \pi$ starting at $(1, 0)$ is

   (a) $(\cos t, \sin t)$ traces out the top half of the circle counter-clockwise starting at $(1, 0)$.

   (b) $(\cos t, -\sin t)$ traces out the bottom half of the circle clockwise starting at $(1, 0)$.

   (c) $(\cos(2t), \sin(2t))$ traces out the complete circle counter-clockwise starting at $(1, 0)$.

   (d) $(\cos(2t), -\sin(2t))$ traces out the complete circle clockwise starting at $(1, 0)$.

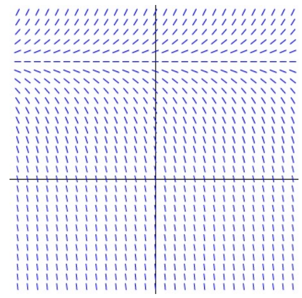
(d) Which differential equation has the direction field shown?

(i) $y'(t) = 6 - 3y$

(ii) $y'(t) = y$

(iii) $y'(t) = 3y - 6$

(iv) $y'(t) = (y-2)e^t$



3. Let R be the region enclosed by the curves $y = 2x$ and $y = x^2$. Write a definite integral that gives the volume of the solid generated by rotating the region R around the line $y = 6$.

4. A solid has a base bounded by the curves $y = x^2$ and $y = 2 - x^2$ for $-1 \le x \le 1$. Cross-sections perpendicular to the x-axis are squares. Write a definite integral for its volume.

5. Compute: (a) $\displaystyle\int x \cos x \, dx$ using integration by parts, (b) $\displaystyle\int \frac{x+1}{x(x-4)} \, dx$ using partial fractions.

6. Use the error bound formulas on the last page to determine an n such that the trapezoid rule with n subintervals approximates $\displaystyle\int_0^1 \frac{1}{e^x} \, dx$ to within .001.

7. (a) Obtain the Taylor series for $\dfrac{1}{1+x}$ at $x = 0$ from the geometric series for $\dfrac{1}{1-x}$.

   (b) Use your result from part (a) and integration to write down the Taylor series at $x = 0$ for $\ln(1+x)$ and then find its interval of convergence.

8. Find the 3rd-order Taylor polynomial centered at 4 for $\dfrac{1}{\sqrt{x}}$.

9. How many terms of $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ do we need to add to estimate the series with $|\text{error}| < 0.001$?

10. Use the integral test to show $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$ and diverges if $0 < p < 1$.

11. Determine which of the following series converges conditionally, converges absolutely or diverges. Specify which convergence test you use and show how it leads to the answer.

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{\ln k}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k^2+50}$ (c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^5}{k!}$ (d) $\displaystyle\sum_{k=0}^{\infty} \frac{5}{2^k+5k+3}$

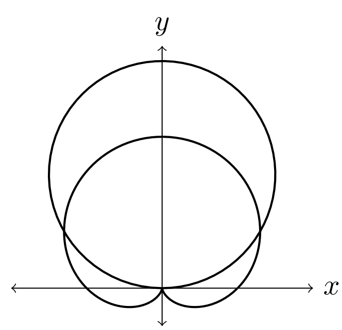
12. Solve for y exactly:

(a) $\dfrac{dy}{dx} = \dfrac{\sin x}{y^2}$ with $y(0) = 3$. (b) $\dfrac{dy}{dx} = y\cos x + xy$ with $y(0) = 3$.

13. Find the orthogonal trajectories of the family of curves $y = kx^4$.

14. A tank contains 60 L of water with 5 kg of salt dissolved in it. Brine that contains 2 kg of salt per liter enters the tank at a rate of 3 L/min. Pure water is also flowing into the tank at a rate of 2 L/min. The solution in the tank is kept well mixed and is drained at a rate of 5 L/min. How much salt remains in the tank after 30 minutes? What happens in the long run?

15. Below are graphs of $r = 3\sin\theta$ and $r = 1 + \sin\theta$.



(a) Determine both polar and rectangular coordinates for all points where the curves cross.

(b) Set up, but do **not** evaluate, an integral for the area of the region inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$.

Midpoint Rule and Error Bound:

$$M_n = (f(\overline{x}_1) + f(\overline{x}_2) + f(\overline{x}_3) + \cdots + f(\overline{x}_n))\Delta x$$

and

$$\left|\int_a^b f(x)\,dx - M_n\right| \leq \frac{K(b-a)}{24}(\Delta x)^2 = \frac{K(b-a)^3}{24n^2},$$

where $\overline{x}_i$ is the midpoint of $[x_{i-1}, x_i]$ and K is an upper bound on $|f''(x)|$ over $[a,b]$: $|f''(x)| \leq K$ for $a \leq x \leq b$.

Trapezoid Rule and Error Bound: Let $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$ with $x_i - x_{i-1} = \Delta x = \dfrac{b-a}{n}$ for all i. Then the nth approximation T_n to $\int_a^b f(x)\,dx$ using the trapezoid rule is

$$T_n = (f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n))\frac{\Delta x}{2}$$

and

$$\left|\int_a^b f(x)\,dx - T_n\right| \leq \frac{K(b-a)}{12}(\Delta x)^2 = \frac{K(b-a)^3}{12n^2},$$

where K is an upper bound on $|f''(x)|$ over $[a,b]$: $|f''(x)| \leq K$ for $a \leq x \leq b$.

Simpson's Rule and Error Bound:

$$S_n = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))\frac{\Delta x}{3}$$

and

$$\left|\int_a^b f(x)\,dx - S_n\right| \leq \frac{K(b-a)}{180}(\Delta x)^4 = \frac{K(b-a)^5}{180n^4},$$

where n is even and K is an upper bound on $|f^{(4)}(x)|$ over $[a,b]$: $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$.