1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

   (a) If the n^{th} partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = 1 + \frac{n}{3^n}$ then $a_n = \frac{2-n}{3^n}$ for $n > 1$: (a) T F

   (b) The geometric series $\sum_{n=4}^{\infty} (\frac{1}{3})^n$ converges to $\frac{3}{2}$. (b) T F

   (c) If $\lim_{n\to\infty} a_n = 0$ then the series $\sum_{n=1}^{\infty} a_n$ converges. (c) T F

   (d) The series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$ converges conditionally. (d) T F

   (e) If $\sum_{n=1}^{\infty} |a_n|$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges. (e) T F

   (f) The sequence $a_n = \frac{\ln(2n)}{\ln(n)}$ converges to 1. (f) T F

   (g) If the power series $\sum_{k=0}^{\infty} a_k\,(x-4)^k$ has a radius of convergence equal to 2 then $\sum_{k=0}^{\infty} a_k$ diverges. (g) T F

2. For each multiple choice question, circle the correct answer. There is only one correct choice for each answer. No justification is required.

   (a) Which of the following sequences is both bounded and monotonic?

   (i) $\{a_n\} = \{n^2\}_0^{\infty}$
   (ii) $\{a_n\} = \{\frac{n}{n+1}\}_0^{\infty}$
   (iii) $\{a_n\} = \{\frac{\sin(\pi n)}{n}\}_0^{\infty}$
   (iv) $\{a_n\} = \{\frac{n}{\sqrt{n+1}}\}_0^{\infty}$
   (v) None of the above

   (b) The value of the telescoping series $\sum_{k=1}^{\infty} (\frac{1}{k} - \frac{1}{k+1})$ is

   (a) 0 (b) 1 (c) 2 (d) 1/2 (e) None of the above

3. Find the centroid for the region bounded by the curves $y = x^2$, the $x = y^2$.

4. Consider the following series, all of which converge. For which of these series do you get a conclusive answer when using the **Ratio Test** to check for convergence? Write the letters of all possible answers. If no series satisfies this condition, write"none". You do not need to show your work.

A $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{2k^5+k^2+1}$ **B** $\displaystyle\sum_{k=1}^{\infty} \frac{k^6}{k!}$ **C** $\displaystyle\sum_{k=1}^{\infty} (3k+4)^{-k}$

D $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$ **E** $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{2}{5^k}$

5. Consider the following sequences and answer the questions that follow by circling all that apply.

(i) $\left\{ a_n = \left(\frac{1-2n}{n+1}\right)^2 \right\}_{n=1}^{\infty}$

(ii) $\{b_n = 3^{n+5}2^{-n}\}_{n=1}^{\infty}$

(iii) $c_n = \left\{ \frac{(-5)^{n+1}}{(3)^n} \right\}$

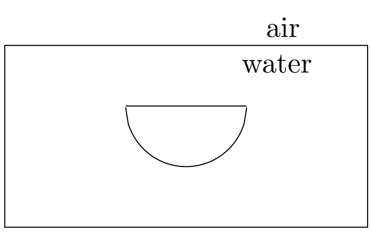
(iv) $S_n = \displaystyle\sum_{k=2}^{n} \frac{k}{k^3-2}$

(a) Which of the above sequences are bounded? (a) a_n (b) b_n (c) c_n (s) S_n [3]

(b) Which of the above sequences are increasing? (a) a_n (b) b_n (c) c_n (s) S_n [3]

(c) Which of the above sequences are convergent? (a) a_n (b) b_n (c) c_n (s) S_n [3]

6. A semi-circular plate with *diameter* 1 m is submerged vertically in water with the diameter being the top of the plate, as in the diagram below. Set up but do **NOT** evaluate an integral equal to the hydrostatic force on the plate, in Newtons, if the top lies 1 meter below the surface of the water. The density of water is 1000 kg/m^3.



7. Determine whether the following series converge conditionally, converge absolutely or diverge. Show your work in applying any tests used.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^2+1}}{k}$ (b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ (c) $\displaystyle\sum_{k=2}^{\infty} k e^{-2k^2}$

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{4+3^k}{4^k}$ (e) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$ (f) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k^2+1}}$

(g) $\displaystyle\sum_{k=1}^{\infty} \frac{k^4}{e^{3k}}$ (h) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k^2-1}$

8. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n n}(x-2)^n$

9. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n} x^n$

(a) Find the radius of convergence, R, for this series.
(b) Find the interval of convergence.
(c) For which x does this series converge absolutely?

10. The infinite series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{2^n}(x-1)^n$ has a radius of convergence R = 2.

(a) Explain carefully what the "radius of convergence" tells us about the series.
(b) Find the interval of convergence.

11. How many terms of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n+5}$ do we need to add in order to find the sum of the series to with an accuracy of 0.00001 (that is, $|\text{error}| < 0.00001$)?