Math 1131 Week 10 Worksheet

Name: ________________________

Discussion Section:_____________

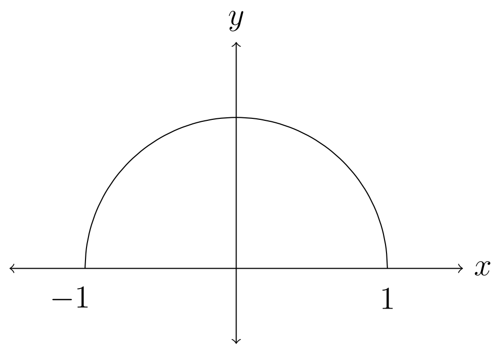
Solutions should show all of your work, not just a single final answer.

4.2: Mean Value Theorem

1. Find every number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^3 - 4x^2 - 5$ on the interval $[1, 2]$.

2. T/F (with justification) The function $1 - \frac{1}{x^4}$ satisfies the hypotheses of Rolle's Theorem on the interval $[-1, 1]$.

3. T/F (with justification) The graph of the semicircle on $[-1, 1]$ below fits the hypotheses of the Mean Value Theorem.



4.3: How Derivatives Affect the Shape of a Graph

4. For the following functions, (i) determine all open intervals where $f(x)$ is increasing, decreasing, concave up, and concave down, and (ii) find all local maxima, local minima, and inflection points. Give all answers **exactly**, not as numerical approximations.

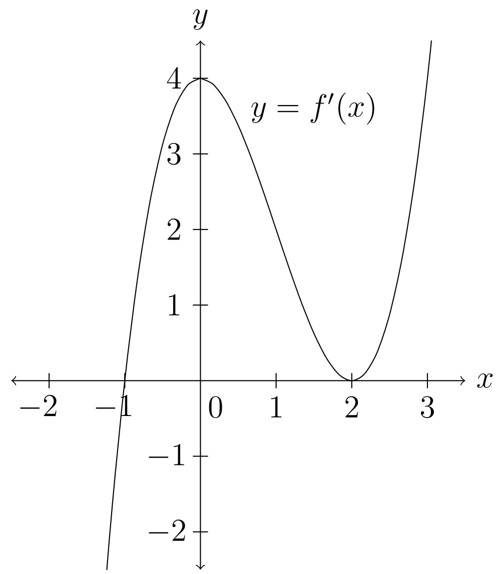
   (a) $f(x) = x^5 - 2x^3$ for all x

   (b) $f(x) = x - 2\sin x$ for $-2\pi < x < 2\pi$

(c) $f(x) = e^{-x} - e^{-3x}$ for $x > 0$

5. For x in the interval $(0, 100)$, let $f(x) = x^{100} + (100 - x)^{100}$. Determine on what open intervals in $(0, 100)$ the function $f(x)$ is increasing and decreasing, and use this information to decide which of $33^{100} + 67^{100}$ or $41^{100} + 59^{100}$ is larger.

6. Below is a graph of $y = f'(x)$ for some function $f(x)$. Determine the intervals where $f(x)$ is increasing and decreasing, the x-values where $f(x)$ has local maxima and minima, and the x-values where $f(x)$ has inflection points.



7. T/F (with justification) If a function $f(x)$ on the interval $(-1, 1)$ is twice differentiable and $f''(c) = 0$ for some c in $(-1, 1)$ then $f(x)$ has an inflection point at $x = c$.

4.4: Indeterminate Forms and l'Hospital's Rule

8. For each of the following limits, indicate what kind of indeterminate form it is and then evaluate it with l'Hospital's rule.

   (a) $\lim_{x \to 0} \dfrac{(x+1)^{11} - 11x - 1}{x^2}$

   (b) $\lim_{x \to 0} \dfrac{\sin(3x)}{e^{9x} - e^{2x}}$

   (c) $\lim_{x \to 0} \dfrac{x - \tan x}{x - \sin x}$

   (d) $\lim_{x \to \infty} \dfrac{\ln(1881x^2 + 1)}{\ln x}$

(e) $\displaystyle\lim_{x\to 0} \frac{\ln(\cos(2x))}{\ln(\cos(3x))}$

(f) $\displaystyle\lim_{x\to\infty} \left(1 + \frac{10}{x}\right)^{x^2}$

(g) $\displaystyle\lim_{x\to 1} \frac{x^x - x}{\ln x}$

(h) $\displaystyle\lim_{x\to 1} \frac{x^x - x^a}{\ln x}$, where a is constant

9. Indicate what kind of indeterminate form $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$ is and then try to evaluate it with l'Hospital's rule. Explain what goes wrong and then evaluate this limit using methods from earlier in the course.