Math 1131 Week 1 Worksheet

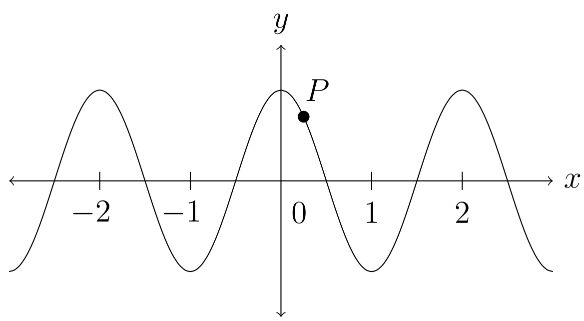
Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

2.1: The Tangent and Velocity Problems

1. The point $P = (1/4, 1/\sqrt{2})$ lies on the curve $y = \cos(\pi x)$ where x is in **radians**, as shown below.



(a) If $Q = (x, \cos(\pi x))$ then use your calculator to find the slope of the secant line PQ, *rounded to four digits after the decimal point*, for the following values of x:

(i) 0.24,

(iv) 0.26,

(ii) 0.249,

(v) 0.251,

(iii) 0.2499,

(vi) 0.2501.

(b) Using the results of part(a), estimate the value of the slope of the tangent line to the curve at $(1/4, 1/\sqrt{2})$ to three digits after the decimal point.

(c) Using the estimated slope from part(b), what is an estimate for the equation of the tangent line to the graph of $y = \cos(\pi x)$ at $(1/4, 1/\sqrt{2})$? Write the final answer in the form $y = mx + b$ where m and b are each rounded to three digits after the decimal point.

2. The displacement of an object on a line, in meters, is $s = 1 + 2t + \frac{1}{4}t^2$, where t is in seconds.

(a) Find the average velocity in m/sec over each of the following time periods. For parts (i) through (v), *round your answer to three digits after the decimal point.* In part (vi) , h is a nonzero variable and the final answer is in terms of h.

(i) $[1, 1.5]$

(ii) $[1, 1.1]$

(iii) $[1, 1.01]$

(iv) $[1, .9]$

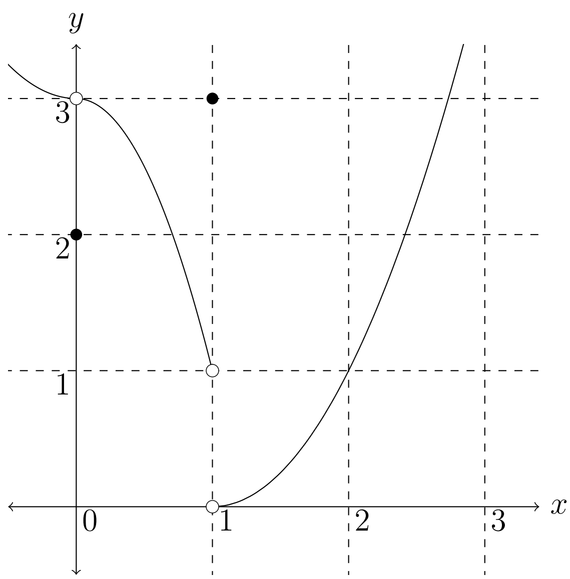
(v) $[1, .99]$

(vi) $[1, 1 + h]$

(b) Use the work in part a to estimate the instantaneous velocity of the object at time $t = 1$, in m/sec.

2.2: The Limit of a Function

3. The graph of $y = f(x)$ is below. Use it to compute each limit or explain why it doesn't exist.



(a) $\lim_{x \to 0^-} f(x)$

(b) $\lim_{x \to 1^-} f(x)$

(c) $\lim_{x \to 2^-} f(x)$

(d) $\lim_{x \to 0^+} f(x)$

(e) $\lim_{x \to 1^+} f(x)$

(f) $\lim_{x \to 2+} f(x)$

(g) $\lim_{x \to 0} f(x)$

(h) $\lim_{x \to 1} f(x)$

(i) $\lim_{x \to 2} f(x)$

(j) $f(0)$

(k) $f(1)$

(l) $f(2)$

4. Determine whether the following limits are finite, ∞, or $-\infty$. If the limit does not exist for any other reason, write DNE with a justification.

   (a) $\lim_{x \to 1} \frac{\sqrt{x}}{2(x-1)^2}$

   (b) $\lim_{x \to 1+} \frac{x-2}{x-1}$

   (c) $\lim_{x \to 0} \frac{1}{x} - \frac{1}{x^2}$

5. T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph of $y = \frac{x^2 - 1}{x^2 - 2x + 1}$.

6. T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph of $y = \frac{x^2 - 2x + 1}{x^2 - 1}$.

2.3: Calculating Limits Using the Limit Laws

7. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leq 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

(a) Sketch the graph of $y = f(x)$ for $-1 \leq x \leq 4$.

(b) Evaluate the following limits if they exist. (If a limit does not exist, write DNE.)

(i) $\lim_{x \to 1^-} f(x)$

(ii) $\lim_{x \to 1^+} f(x)$

(iii) $\lim_{x \to 1} f(x)$

(iv) $\lim_{x \to 2^-} f(x)$

(v) $\lim_{x \to 2^+} f(x)$

(vi) $\lim_{x \to 2} f(x)$

8. Evaluate the following limits exactly using algebra and limit laws (some limits may be DNE).

(a) $\lim_{x \to 2} \frac{x^3 - 2}{2x^2 - 3x + 2}$

(b) $\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$

(c) $\lim_{x \to 3} \frac{\sqrt{x^2 + 40} - 7}{x - 3}$

(d) $\lim_{x \to -2} \sqrt{x^4 + 3x + 6}$

(e) $\lim_{x \to 1} \frac{x^2 + 4x}{x^2 + 3x - 4}$

(f) $\lim_{x \to 1} \frac{(x^2 + x)^2 - 4}{x^2 + x - 2}$

9. Evaluate the following limits using algebra and limit laws (some limits may be DNE). Note that a represents a constant, and answers may be in terms of a.

   (a) $\lim_{t\to 0} \frac{\sqrt{a+t}-\sqrt{a-t}}{t}$ for $a > 0$

   (b) $\lim_{h\to 0} \frac{1/(a+h)^2 - 1/a^2}{h}$ for $a \neq 0$

10. T/F (with justification) If $\lim_{x\to 2} g(x) = 0$ and $\lim_{x\to 2} h(x) = 0$ then $\lim_{x\to 2} \frac{g(x)}{h(x)}$ does not exist.