Section 2.6: Limits at Infinity; Horizontal Asymptotes

(1) What does it mean to take the limit of a function as $x \to \infty$ or $x \to -\infty$?

> We want to determine what the function value is approaching as x increases more and more. (Or decreases more and more for $x \to -\infty$

(2) What is the definition of a horizontal asymptote? Can a function cross its horizontal asymptote? Make sure you know the difference between a horizontal and vertical asymptote and the limit definition of each.

> If $\lim_{x\to\infty} f(x) = a$ or $\lim_{x\to-\infty} f(x) = a$ (where a is a finite number), then $f(x)$ has a HA at $x = a$. A function CAN cross its horizontal asymptote (infinitely many times, if it wants). We get a HA when we take the limit as x goes to $\pm\infty$. We get a vertical asymptote when the limit (or one sided limit) as x goes to a equals $\pm\infty$.

(3) What algebraic technique do we use to evaluate limits of rational functions at ∞? Note: this techniques should only be used as $x \to \infty$ or $x \to -\infty$. For other functions, we can also multiply (top and bottom) by the conjugate or use the Squeeze Theorem.

> We should divide by x to its highest power. (when determining the power of an x under a sqaure root, be sure to take the sqaure root into consideraion.)

(4) When we have a square root function and we are dividing by x^2, we have to be careful since $\sqrt{x^2} = |x| \neq x$ when $x < 0$. Show an example where this comes up.

If we want to compute $\lim_{x\to-\infty} \frac{\sqrt{4x^2+x}}{4x}$, we notice the highest power of x is x^1. (note: there is an x^2 but since its under the squareroot, it counts as x^1.) So we divide top and bottom by x. This gives:

$$\lim_{x\to-\infty} \frac{\frac{\sqrt{4x^2+x}}{x}}{\frac{4x}{x}}$$

Simplifying the bottom gives us:

$$\lim_{x\to-\infty} \frac{\frac{\sqrt{4x^2+x}}{x}}{4}$$

To simplify the numerator, we want to write $x = \sqrt{x^2}$, but in this case $x \to -\infty$, so $x < 0$. Since $\sqrt{x^2} = |x|$, we instead get that

$$x = -\sqrt{x^2}.$$

So we get

$$\lim_{x\to-\infty} \frac{\frac{\sqrt{4x^2+x}}{-\sqrt{x^2}}}{4}$$

Now, we can combine the square roots

$$\lim_{x\to-\infty} \frac{-\sqrt{\frac{4x^2+x}{x^2}}}{4}$$

Now, we can separate the numerator of the numerator:

$$\lim_{x\to-\infty} \frac{-\sqrt{\frac{4x^2}{x^2} + \frac{x}{x^2}}}{4} = \lim_{x\to-\infty} \frac{-\sqrt{4 + \frac{1}{x}}}{4}$$

As $x \to -\infty$, $1/x \to 0$, so we get

$$\frac{-\sqrt{4}}{4} = -1/2,$$

as the limit.

(5) What is $\lim_{x\to\infty} f(x)$? for $f(x) = \ln(x), e^x, \sin(x), \cos(x), x^n$.

$$\lim_{x\to\infty} \ln(x) = \infty$$

$$\lim_{x\to\infty} e^x = \infty$$

$\lim_{x\to\infty} \sin(x)$ does not exist since $\sin(x)$ oscillates between -1 and 1.

$\lim_{x\to\infty} \cos(x)$ does not exist since $\cos(x)$ oscillates between -1 and 1.

$$\lim_{x\to\infty} x^n = \infty \text{ for } n > 0$$

$$\lim_{x\to\infty} x^n = 1 \text{ for } n = 0$$

$$\lim_{x\to\infty} x^n = 0 \text{ for } n < 0$$

Extra Practice in Book: 2.6: 1, 3, 9, 19, 23, 24, 27, 38, 47