Section 2.5: Continuity

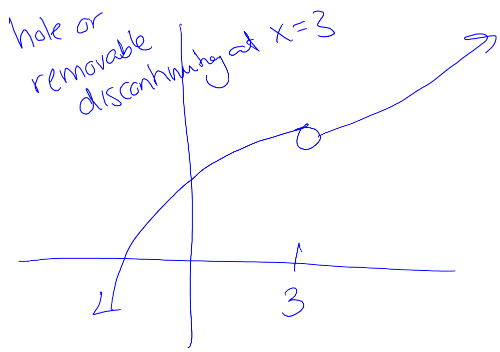
(1) What is the definition of a function being continuous at a number a? What three things to to be true this to happen?

> A function $f(x)$ is continuous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$. In order for this to happen, (1) $\lim_{x\to a} f(x)$ must exist, (2) $f(a)$ must be defined and (3) they must be equal.

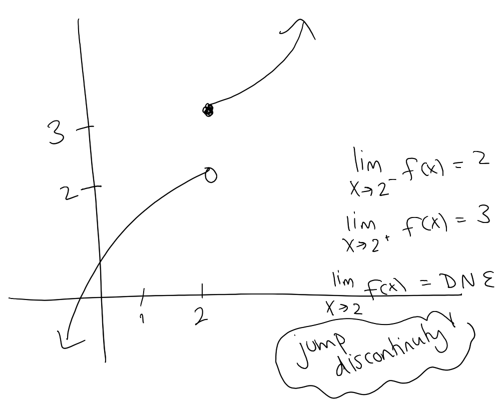
(2) For each of the three ways to fail above we get a different type of discontinuity.

(a) If the function is not continuous at $x = a$ because $f(a)$ is undefined, what type of discontinuity do we get? Illustrate with a graph.

> We get a hole.

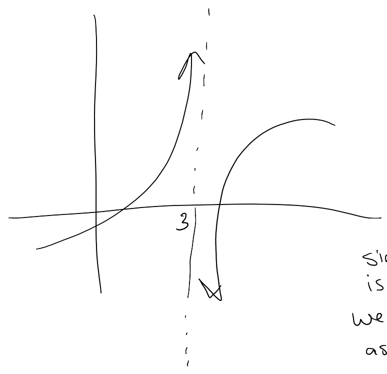


(b) If the function is not continuous at $x = a$ because the limit does not exist because the left hand and right hand limits are not equal, what type of discontinuity do we get? Illustrate with a graph.



> We get a jump discontinuity.

(c) If the function is not continuous at $x = a$ because the limit does not exist because (at least) one side approaches infinity, what type of discontinuity do we get? Illustrate with a graph.



$\lim_{x \to 3^-} f(x) = +\infty$

$\lim_{x \to 3^+} f(x) = -\infty$

Since (at least one) is $+\infty$ or $-\infty$ we get a vertical asymptote at $x=3$

We get a vertical asymptote.

(3) What does it means for a function to be continuous from the left at $x = a$? Continuous from the right? How can you tell if a piecewise function is continuous from the left or right?

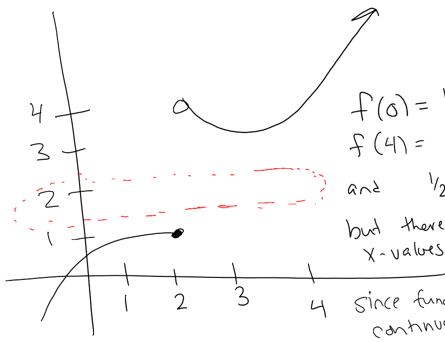
A function is continuous from the left if the left hand limit of the function equals the function value. It is continuous from the right if the right hand limit of the function equals the function value. To tell if a piecewise function is continuous from the left (or right), you need to evaluate the one sided limit and compare that to the function value.

(4) For non-piecewise defined functions, the standard ones are defined at every point in their domain. The big three things we need to remember is that we can't divide by 0, can't take the square (or other even) root of a negative number and can't take the log of 0 or a negative. For piecewise functions, you need to check that the functions meet up where the functions change. Write a few sentences about how to check if a piecewise function is continuous.

First, you need to check if the different functions that make up the piecewise function are continuous where they are used. Then we need to check the places in the domain where you switch from one function to the other to see if they meet up. To do this, find the x value where the functions change and plug that into each of the function that are valid on either side and make sure they are the same. If the function value is defined separately, make sure that value is equal to the limit from the left and right. Do this for each x value where you change the functions.

(5) What does the Intermediate Value Theorem say? What are the conditions that need to be met in order to reach the conclusion of IMT? Draw a picture of a function that does not satisfy the conclusion of the IMT because it is not continuous.

> The Intermediate Value Theorem says if you have a continuous function that is equal to y_1 at $x = a$ and y_2 at $x = b$ then it also must equal to every value between y_1 and y_2 between $x = a$ and $x = b$. The function needs to be continuous on $[a, b]$ in order for this to apply.



$f(0) = 1/2$
$f(4) = 4$
and $1/2 \leq 2 \leq 4$
but there are no x-values so that $f(x) = 2$.
since function is not continuous.

Extra Practice in Book: 2.4:1, 3, 7, 19, 23, 43, 46, 50, 52