Section 2.3: Calculating Limits Using Limit Laws

(1) There are five limit laws presented at the beginning of the sections. What are they? Write them in your own words.

> When evaluating limits we can break them up over addition, subtraction, multiplication and division* and we can pull out constants.

(2) One of the limit laws has a special constraint. What is it?

> When we are finding the limit of a quotient of functions, we need to make sure the function in the denominator is not approaching 0.

(3) Read through rules 6-11. Make some notes about these rules.

> We can switch the order of limits and (positive integer) exponents, the limit as $x \to a$ of c is c and of x is a. Can also just plug in $x = a$ for x^n and $\sqrt[n]{x}$. And can interchange the nth root of a function with the limit.

(4) The main idea between all of these rules is that as long as the function is continuous, you can evaluate each of the parts separately (like splitting up the sum) or apply the limit first and then do the operation (like taking the power outside the limit). See the direct substitution property.

> It says that is if f is a polynomial or rational function and we want to evaluate its limit as $x \to a$, then as long as a is in the domain, we can just plug in $x = a$.

(5) There are several algebraic techniques to solve limits, including factoring, expanding, multiplying by the conjugate and finding a common denominator.

(a) Give an example of a limit solved by factoring.

> Many possible answers. One is $\lim_{x\to 3} \dfrac{x^2 + 2x - 15}{x - 3}$

(b) Give an example of a limit solved by expanding.

> Many possible answers. One is $\lim_{x\to 2} \dfrac{(x+2)^2 - 16}{x - 2}$

(c) Give an example of a limit solved by multiplying by the conjugate.

Many possible answers. One is $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

(d) Give an example of a limit solved by finding a common denominator.

Many possible answers. One is $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

(6) Sometimes, especially with piecewise functions, you find the limit by considering the two one sided limits. Explain how this works with an example.

Let $f(x) = \begin{cases} x+2 & x<0 \\ 7 & x=0 \\ x^2+2 & x>0 \end{cases}$

If we want to find $\lim_{x\to 0} f(x)$, we should consider $\lim_{x\to 0-} f(x)$ and $\lim_{x\to 0+} f(x)$ since the function definition switches at 0. To find $\lim_{x\to 0-} f(x)$, we only need to consider the function $x+2$ since thats the function that is valid to the left of 0. It is approaching 2, so $\lim_{x\to 0-} f(x) = 2$. Now for $\lim_{x\to 0+} f(x)$, we consider x^2+2 since that is what is valid for $x>0$. As $x\to 0$, this function also approaches 2, so we get $\lim_{x\to 0+} f(x) = 2$. Now since both one-sided limits approach 2, we can say that the limit itself approaches 2. So $\lim_{x\to 0} f(x) = 2$. Note, this is all completely independent of $f(2)$.

(7) Consider $f(x) = x^2\sin(\frac{1}{x})$. Sketch a graph of this function, x^2 and $-x^2$. (Use Desmos or GeoGebra as online graphers). Then explain algebraically why $f(x) = x^2\sin(\frac{1}{x})$ is between x^2 and $-x^2$. Then explain how to use the squeeze theorem to find the limit.

See the figure above. Note that

$$-1 < \sin(x) < 1$$

for all x. So plugging in $1/x$ this property still holds. So we have:

$$-1 < \sin(\frac{1}{x}) < 1$$

Now multiplying through by x^2 (which is positive so doesn't affect the inequality gives us:

$$-x^2 < x^2\sin(\frac{1}{x}) < x^2$$

Since we have squeezed $x^2\sin(1/x)$ between $-x^2$ and x^2 (compare graph to inequality), and since $\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$, we can conclude by the squeeze theorem that $\lim_{x\to 0} x^2\sin(1/x) = 0$.

Important ideas to know from this section:

- In the limit laws, we have to be careful when dividing if we are dividing by something that tends to 0.
- The algebraic techniques of limits including direct substitution, factoring, expanding, multiplying by the conjugate and finding a common denominator will be used often. Make sure you are familiar with them.

Extra Practice in Book: 2.3:1, 5, 11, 17, 21, 23, 37, 51