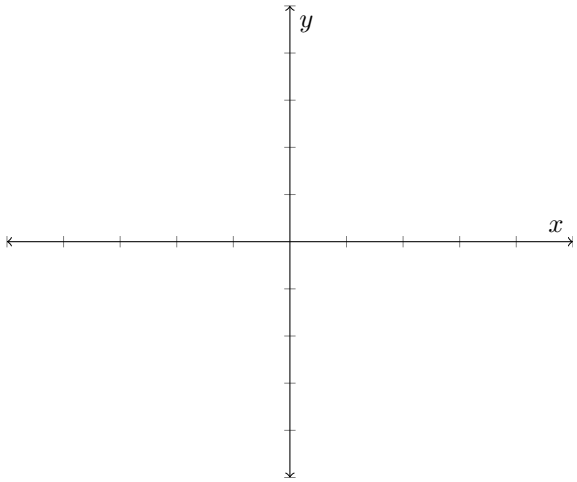
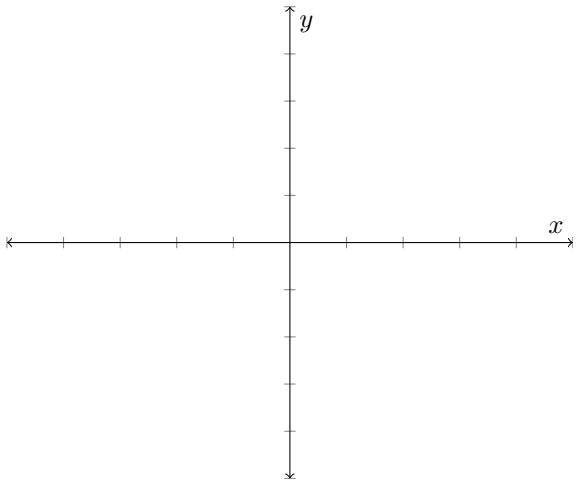
Graph Transformations

If you know what the graph of $f(x) = 3\sqrt{x^5 - 4x}$ looks like, then it's a good bet that the graph of $f(x) = 3\sqrt{x^5 - 4x} + 1$ is related. If you took that bet, you would win. We'll talk about graph transformations in this worksheet.

1. (a) Sketch a graph of $f(x) = x^2$.



(b) Sketch a graph of $g(x) = x^2 + 1$.



(c) How are the two related? Why does this make sense?

You could view the above "modification" to the function $f(x) = x^2$ as saying $g(x) = f(x)+1$. That is, we just figured out how the graphs of $f(x)$ and $f(x) + 1$ are related.

2. How are the graphs of $f(x)$ and $f(x) - 5$ related?

Let's continue to look at modifications we can make to a function that affect the *output* of the function.

3. What's the difference between $f(x) = x^2$ and $g(x) = 2x^2$? (That is, the difference between $f(x)$ and $2f(x)$?)

4. What's the difference between $f(x) = x^2$ and $g(x) = \frac{3x^2}{4}$? (That is, the difference between $f(x)$ and $\frac{3}{4}f(x)$?)

5. In general, to get the graph of $cf(x)$, we can take the graph of $f(x)$ and _____

   _____.

6. What's the difference between $f(x) = x^2$ and $g(x) = -x^2$?

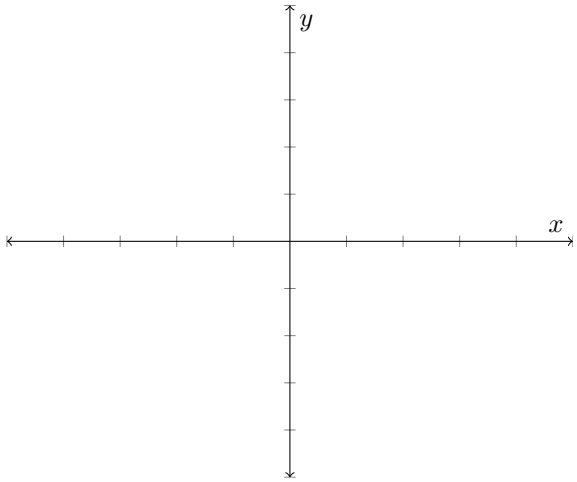
7. In general, to get the graph of $-f(x)$, we can take the graph of $f(x)$ and _____

   _____.

Notice that all of these transformations affected the *output* of the function. That is, they all involved computing $f(x)$, getting the output of the function, and *then* adding 3 or multiplying by 17 or doing something to that result. However, we can also modify the *input* to a function.

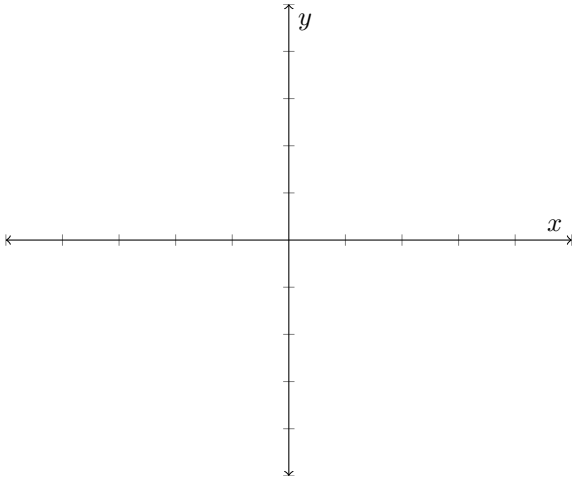
8. (a) Let $f(x) = x^2$. What is the function $g(x) = f(x - 1)$? Simplify.

   (b) Sketch a graph of f and g on the same axes:



(c) What is the relationship between the graph of $f(x)$ and the graph of $f(x-1)$?

9. What is the relationship between the graph of $f(x)$ and the graph of $f(2x)$? Here are some axes to use in case you want to try some examples.



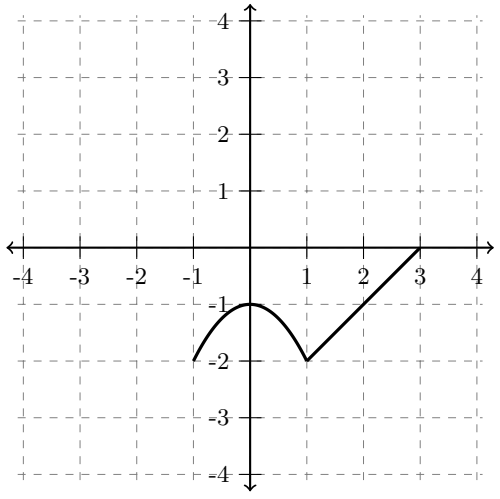
10. What about between $f(x)$ and $f(\frac{x}{2})$?

11. What about between $f(x)$ and $f(-x)$? (Why isn't $f(x) = x^2$ a good example for this one?)

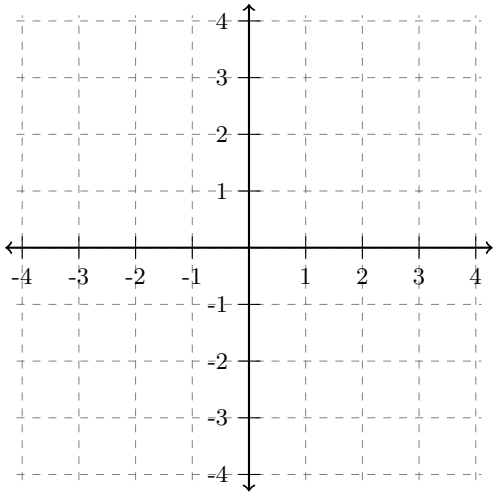
12. Can you summarize your results to all of the above?

Now give it a try in a case where you don't know the formula for the function. Remember, functions can be represented with a formula, or graphically, or even in tables; to define a function all we need is inputs and outputs. Although sometimes one form is more user-friendly than another, depending on the situation, now that you have graph transformations you can do more with graphical interpretations.

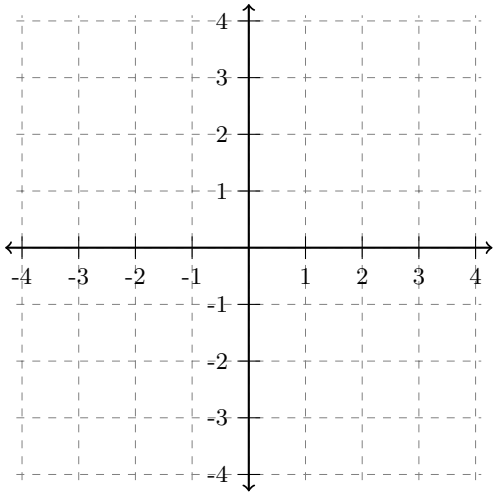
13. The graph of a function f is shown.



(a) Sketch a graph of $f(x+2)+2$.



(b) Sketch a graph of $2f(x)$.



(c) If $g(x) = -3f(3-2x)+5$, what is $g(2)$? (Hint: don't bother graphing it, it's not necessary. Although you could graph it if you wanted to...)

14. When you get stuck on graph transformations, if you forget how they work, how could you go about figuring it out again, or at least checking your answers?