

Name: _____

Score: _____ /20

Understanding Surfaces

Please staple your work and use this page as a cover page.

- Consider the sphere $x^2 + y^2 + z^2 = 8$, and the half cone $z = \sqrt{x^2 + y^2}$ (the full cone is $z^2 = x^2 + y^2$, and the other half is $z = -\sqrt{x^2 + y^2}$). Find the intersection of these two surfaces—what kind of curve is it? can we say how large it is?
 - Find the equation of a sphere whose intersection with the half cone $z = \sqrt{x^2 + y^2}$ is a single point.
 - Find the equation of a sphere that does not intersect the half cone $z = \sqrt{x^2 + y^2}$.
- For each of the following, describe all possible intersections of the given surfaces. If it is possible for the surfaces to not intersect, be sure to mention this as well.
 - Two planes
 - A plane and a circular cylinder
 - A plane and a sphere
 - Two spheres
 - An elliptic paraboloid and a plane
 - A (true) cone and a plane
- What is the difference between $x^2 + y^2 = 9$ and $x^2 + y^2 \leq 9$? What do they look like? Do they both have surface area? volume?
- Sketch the region in \mathbb{R}^3 given by $1 \leq x^2 + y^2 \leq 4$, $z \leq 0$.
- Assume that gravity acts in the direction of the negative z -axis. If we poured water from high above the xy -plane, which of the surfaces $z = x^2$, $z = y^2$, $x = y^2$, and $y = z^2$ would hold the water? Why or why not?