Name: ______________________

Score: ________ /20

Tangent Planes and Directional Derivatives

Please staple your work and use this page as a cover page.

1. Find an equation of the tangent plane for $z = x\sin(x+y)$ at $(-1, 1)$.

2. Consider the function $f(x, y) = \dfrac{2x+3}{4y+1}$.
   (a) Find an equation of the tangent plane to the surface $z = f(x, y)$ at $(0, 0)$.
   (b) Use your equation from part (a) to approximate the value of $f(0.01, 0.01)$, and find the actual value of $f(0.01, 0.01)$ rounded to three decimal places to compare.

3. We don't know the equation that defines a certain surface S, but we are able to determine two tangent vectors at the point $(2, 1, 3)$ on the surface, namely

$$\vec{v}_1 = \langle 3, 0, -4\rangle \qquad \text{and} \qquad \vec{v}_2 = \langle 1, 6, 2\rangle.$$

   Using this information, determine an equation for the tangent plane at $(2, 1, 3)$.

4. The function $A(x, y) = 4000 + 3xy - 4x^2 - 5y^2$ gives the altitude in feet at any point (x, y) on a hill (we can think of the (x, y) coordinates as essentially giving longitude and latitude). We are currently located on the hill at $(-1, 2)$.
   (a) Find our current altitude.
   (b) Find the initial slope if we start moving in the direction of the vector $\langle 1, 7\rangle$
   (c) Find a vector that points in a direction in which the initial slope will be 0 (your vector does not have to be unit).

5. Let $f(x, y) = 8e^{y\sqrt{x}} - x^2y^3$.
   (a) Find the derivative of f in the direction $\langle 5, 3\rangle$ at the point $(4, -1)$.
   (b) At the point $(4, -1)$, find the direction in which the maximum derivative of f occurs, and find the maximum derivative of f.