

Name: _____

Score: _____ /20

Double Integrals in Polar Coordinates

Please staple your work and use this page as a cover page.

1. Let D be the region in the first quadrant of the xy -plane given by $1 \leq x^2 + y^2 \leq 4$. Set up and evaluate a double integral of the function $f(x, y) = xy$ over the region.

2. Evaluate each of the following double integrals by converting to polar coordinates.

(a)
$$\int_{-1}^0 \int_0^{\sqrt{1-x^2}} 8x^3y \, dy \, dx$$

(b)
$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} e^{x^2+y^2} \, dx \, dy$$

3. Let D be the region in the third quadrant enclosed by $y = x$, $y = 0$, and $x^2 + y^2 = 16$.

(a) Sketch the region D .

(b) Write the double integral $\iint_D \frac{3x^2}{y} \, dA$ in polar coordinates (do not evaluate).

4. Suppose we want to find the volume between the planes $z = x - y$ and $z = 0$ inside the cylinder $x^2 + y^2 = 4$.

(a) Evaluate $\iint_D x - y \, dA$ where D is the region $x^2 + y^2 \leq 4$. Why is this not the desired volume?

(b) Try to sketch the volume we are looking for: sketch the plane $z = x - y$, then the cylinder $x^2 + y^2 = 4$. Now use symmetry and a double integral to compute the volume described above.