

Name: _____

Score: _____ /20

Line Integrals vs. Surface Integrals

Please staple your work and use this page as a cover page.

1. Let $\vec{F} = \langle e^{yz}, xze^{yz}, xye^{yz} \rangle$.

(a) Show that \vec{F} is a conservative vector field.

(b) If C is the curve of intersection between the cylinder $x^2 + y^2 = 4$ and the plane $3x - 2y + 7z = 12$ (what is the shape of this intersection?), evaluate the line integral

$$\int_C \vec{F} \cdot d\vec{r}.$$

(c) If S is the portion of the xz -plane with $-1 \leq x \leq 1$ and $0 \leq z \leq 1$ oriented in the direction of the y -axis, evaluate the surface integral

$$\iint_S \vec{F} \cdot d\vec{S}.$$

(d) If S is the surface $z = \sqrt{x^2 + y^2}$ with $0 \leq z \leq 4$ and outward orientation, evaluate the surface integral

$$\iint_S \text{curl} \vec{F} \cdot d\vec{S}.$$

2. Let $\vec{F} = \langle -y, z, -x \rangle$.

(a) If C is the circle $x^2 + y^2 = 4$, $z = 0$, traversed once counterclockwise, evaluate the line integral

$$\oint_C \vec{F} \cdot d\vec{r}.$$

(b) If S is the surface $x^2 + y^2 \leq 4$, $z = 0$, with upward orientation, evaluate the surface integral

$$\iint_S \vec{F} \cdot d\vec{S}.$$

(c) Compute $\text{curl} \vec{F}$.

(d) If S is the portion of the surface $z = 4 - x^2 - y^2$ with $z \geq 0$ and upward orientation, evaluate the surface integral

$$\iint_S \text{curl} \vec{F} \cdot d\vec{S}.$$