

Name: _____

Score: _____ /20

Line Integral Theorems

Please staple your work and use this page as a cover page.

1. Let $\vec{F}(x, y, z)$ be a vector field and assume $\vec{F} = \vec{\nabla}f$ for some function $f(x, y, z)$. If $\int_{C_1} \vec{F} \cdot d\vec{r} = 2$ and C_1 is given by $\vec{r}_1(t) = \langle t, t^2, t^3 \rangle$, $0 \leq t \leq 1$, then what is the value of $\int_{C_2} \vec{F} \cdot d\vec{r}$, if C_2 is given by $\vec{r}_2(t) = \langle 1 - \frac{1}{2}t, -\frac{1}{8}(t^3 - 8), \cos(\frac{\pi t}{4}) \rangle$, $0 \leq t \leq 2$? Justify your answer. Hint: think about where C_1 starts and ends, and where C_2 starts and ends.

2. Let $\vec{F}(x, y) = \langle y^2, x^2y \rangle$ and C the rectangle with vertices $(0, 0)$, $(5, 0)$, $(5, 4)$, and $(0, 4)$ and positive orientation. Consider the line integral

$$\int_C \vec{F} \cdot d\vec{r}.$$

- (a) Compute this line integral directly.
(b) Compute this line integral using Green's Theorem.

3. Compute $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = \langle y \cos x - xy \sin x, xy + x \cos x \rangle$ and C is the triangle formed by going from $(0, 0)$ to $(0, 4)$ to $(2, 0)$ to $(0, 0)$.

4. Suppose we have a region D in the plane with closed boundary curve C that is positively oriented, \vec{n} the outward unit normal vector to the curve C , and a vector field $\vec{F} = \langle P, Q \rangle$. The following form of Green's Theorem says we can determine the outward flow of \vec{F} across C (this is what integrating $\vec{F} \cdot \vec{n}$ measures) by evaluating a double integral over the enclosed region D .

$$\int_C \vec{F} \cdot \vec{n} \, ds = \iint_D \operatorname{div} \vec{F} \, dA = \iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) \, dA,$$

Using this version of Green's Theorem, compute $\int_C \vec{F} \cdot \vec{n} \, ds$, if $\vec{F} = \langle xy^2, x^2y \rangle$ and C is the curve given by $\vec{r}(t) = \langle 3 \cos t, 3 \sin t \rangle$, $0 \leq t \leq 2\pi$ with positive orientation.