

Name: \_\_\_\_\_

Score: \_\_\_\_\_ /20

# Using the Divergence Theorem

Please staple your work and use this page as a cover page.

1. Let  $\vec{F} = \langle z, y, x \rangle$  and let  $S$  be the surface  $x^2 + y^2 + z^2 = 16$  with outward orientation.

(a) Compute the flux of  $\vec{F}$  across the surface  $S$  using the **definition** of a surface integral.

You can use the parametrization  $\vec{r}(u, v) = \langle 4 \sin v \cos u, 4 \sin v \sin u, 4 \cos v \rangle$ , but make sure you understand where this comes from and could produce it yourself if necessary. Also free to use the following:

$$\vec{r}_u \times \vec{r}_v = \langle 16 \sin^2 v \cos u, 16 \sin^2 v \sin u, 16 \sin v \cos v \rangle.$$

(b) Use the Divergence Theorem to find the flux, and make sure your answer agrees with part (a).

2. Let  $S$  be the surface of the solid bounded by  $y^2 + z^2 = 1$ ,  $x = -1$ , and  $x = 2$  and let  $\vec{F} = \langle 3xy^2, xe^z, z^3 \rangle$ . Calculate the flux of  $\vec{F}$  across the surface  $S$ , assuming it has positive orientation.

3. Let  $S$  be the surface  $x^2 + y^2 + z^2 = 4$  with positive orientation and let  $\vec{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$ . Evaluate the surface integral

$$\iint_S \vec{F} \cdot d\vec{S}.$$

4. Let  $\vec{F} = \langle P, Q, R \rangle$  be a vector field and  $S$  a closed surface with positive orientation containing a solid  $E$ . Use the Divergence Theorem to compute

$$\iint_S \text{curl} \vec{F} \cdot d\vec{S}.$$

5. Compute  $\iint_S \vec{F} \cdot d\vec{S}$  if  $\vec{F} = \langle x^2y, -xy^2, 4z(z^2 - 1) \rangle$  and  $S$  is the unit cube with outward orientation with vertices at  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(1, 1, 0)$ ,  $(0, 0, 1)$ ,  $(1, 0, 1)$ ,  $(0, 1, 1)$ , and  $(1, 1, 1)$ , minus the top face (so  $S$  is like a box with no lid). Notice that  $S$  is not closed. How might we still be able to make use of the Divergence Theorem?