Triple Integrals

To integrate a function f over a three-dimensional region (a solid) E, we use a triple integral:

$$\iiint_E f\, dV$$

Cartesian coordinates: (x, y, z)

- $dV = dx\, dy\, dz$ in some order

- type x: one bounding surface on back, one on front
type y: one bounding surface on left, one on right
type z: one bounding surface on bottom, one on top

- If the solid E is type x, we can integrate over it using one triple integral with the innermost integral with respect to x. Analogous for types y and z.

- Examples of generic type x, y, and z solids are shown below:

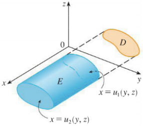


FIGURE 7

type x

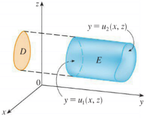


FIGURE 8

type y

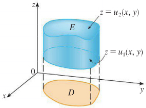
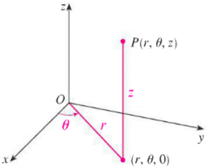


FIGURE 2

type z

Cylindrical coordinates: (r, θ, z)



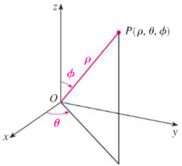
$x = r\cos\theta$
$y = r\sin\theta$
$z = z$
$x^2 + y^2 = r^2$

$dV = r\, dz\, dr\, d\theta$
(in some order)

- Good for: cylinders, cones, paraboloids

- Can handle if needed: spheres

Spherical coordinates: (ρ, θ, ϕ)



$x = \rho\sin\phi\cos\theta$
$y = \rho\sin\phi\sin\theta$
$z = \rho\cos\phi$
$x^2 + y^2 + z^2 = \rho^2$

$dV = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$
(in some order)

- Good for: spheres, cones

- Can handle if needed: cylinders