Computing Surface Integrals

Are we integrating a **scalar function** f or a **vector field** $\vec{F}$?

scalar function $\longrightarrow$ $\displaystyle\iint_S f\,dS = \iint_D f(\vec{r}(u,v))\,|\vec{r}_u \times \vec{r}_v|\,dA$

can be used for any scalar function

vector field $\downarrow$

Is S a **closed surface** enclosing a solid E?

no $\longrightarrow$ $\displaystyle\iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(u,v))\cdot(\vec{r}_u \times \vec{r}_v)\,dA$

can be used for any vector field

yes $\downarrow$

Divergence Theorem:
$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}(\vec{F})\,dV$$

If we want to integrate $\operatorname{curl}(\vec{F})$ across an oriented surface S with boundary curve C that is traced counterclockwise when viewed from the positive side of S,

Stokes' Theorem: $\displaystyle\iint_S \operatorname{curl}(\vec{F})\cdot d\vec{S} = \int_C \vec{F}\cdot d\vec{r}$