Exam 3 Review Problems

1. Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$$

2. Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{x}{2x^2+1}$$

3. Find the Maclaurin series for the given function.

$$f(x) = x^2 \ln(1+x^3)$$

4. Let

$$f(x) = \sin x, \quad a = \pi/6, \quad n = 4, \quad 0 \le x \le \pi/3.$$

   (a) Approximate f by a Taylor polynomial with degree n at number a.

   (b) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.

5. Find the length of the arc of the curve from point P to point Q.

$$x^2 = (y-4)^3, \quad P(1,5), \quad Q(8,8)$$

6. The given curve is rotated about the y-axis. Find the area of the resulting surface.

$$y = 1 - x^2, \quad 0 \le x \le 1$$

7. Let

$$x = \frac{1}{2}\cos\theta, \quad y = 2\sin\theta, \quad 0 \le \theta \le \pi.$$

   (a) Eliminate the parameter to find a Cartesian equation of the curve.

(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

8. Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = t - t^{-1}, \quad y = 1 + t^2; \quad t = 1$$

9. Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

$$x = \cos 2t, \quad y = \cos t, \quad 0 < t < \pi$$

10. The Cartesian coordinates of a point are given. (i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$. (ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.

   (a) $(3\sqrt{3}, 3)$

   (b) $(1, -2)$

11. Find the points on the given curve where the tangent line is horizontal or vertical.

$$r = e^{\theta}$$