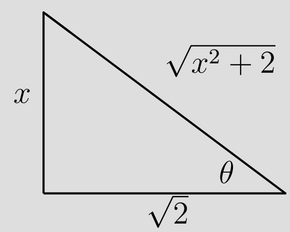
Exam 1 Review T/F

1. T/F (with justification): The value of $\int_{-\pi}^{\pi} \sin^9 x\, dx$ is 0.

   Solution: TRUE. The function $\sin^9 x$ is odd and we are integrating over an interval of the form $[-b, b]$ with $b = \pi$.

2. T/F (with justification): To evaluate $\int \frac{dx}{x^2\sqrt{x^2+2}}$ by trigonometric substitution, use $x = 2\tan\theta$.

   Solution: FALSE. We want a triangle with hypotenuse $\sqrt{x^2+2}$. To write $\sqrt{x^2+2}$ as $\sqrt{x^2+a^2}$, we want $a = \sqrt{2}$.

   This picture shows the triangle and the correct trigonometric substitution is $x = \sqrt{2}\tan\theta$, not $x = 2\tan\theta$.

3. T/F (with justification): For differentiable $f(x)$,

$$\int_0^{\pi} f(x)\cos x\, dx = -\int_0^{\pi} f'(x)\sin x\, dx.$$

   Solution: TRUE.

   (1) Set $u = f(x)$ and $dv = \cos x\, dx$.

   (2) We have $du = f'(x)\, dx$ and $v = \sin x$.

(3) By integration by parts,

$$\int_0^\pi f(x)\cos x\,dx = \int_0^\pi u\,dv = uv\Big|_0^\pi - \int_0^\pi v\,du = f(x)\sin x\Big|_0^\pi - \int_0^\pi f'(x)\sin x\,dx.$$

Since $\sin\pi = 0$ and $\sin 0 = 0$, the first term on the right is $0 - 0 = 0$, so

$$\int_0^\pi f(x)\cos x\,dx = -\int_0^\pi f'(x)\sin x\,dx.$$

4. T/F (with justification) Computing $\int \frac{x}{x^2-1}\,dx$ requires partial fractions.

Solution: FALSE. The substitution $u = x^2 - 1$ can be used: $du = 2x\,dx$, so

$$\int \frac{x}{x^2-1}\,dx = \int \frac{1/2}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2-1| + C.$$

5. T/F (with justification) The Trapezoidal Rule for $\int_a^b f(x)\,dx$ has no error if $f(x)$ is linear.

Solution: TRUE. By a *picture*, all the trapezoidal pieces line up to give one big trapezoid, since $f(x)$ is linear, and this trapezoid is exactly the region whose signed area is $\int_a^b f(x)\,dx$.

6. T/F (with justification) The integral $\int_0^2 \frac{dx}{x-1}$ is convergent.

Solution: FALSE. The integrand $1/(x-1)$ blows up at 1, which is in $[0,2]$, so we have to treat $\int_0^2$ as $\int_0^1 + \int_1^2$, and check both of these integrals converge. Since

$$\int_1^2 \frac{dx}{x-1} = \lim_{a\to 1^+}\int_a^2 \frac{dx}{x-1} = \lim_{a\to 1^+} -\ln(a-1) = \infty,$$

the integral is not convergent.

Note: Without realizing there's a problem with the integrand inside $[0, 2]$, a common mistake would be to say the integral is $\ln|x-1|\big|_0^2 = \ln 1 - \ln 1 = 0$ and then say the integral "converges." Remember: proper integrals $\int_a^b f(x)\,dx$ require the integrand $f(x)$ to be continuous on the *entire* interval $[a, b]$.