Math 1132 Worksheet 8.2 Name: _____ Discussion Section: _____

8.2 Area of a Surface of Revolution

Surface Area of a Surface. The surface area of a surface obtained by rotating the curve $y = f(x)$, $a \leq x \leq b$, about the x-axis is

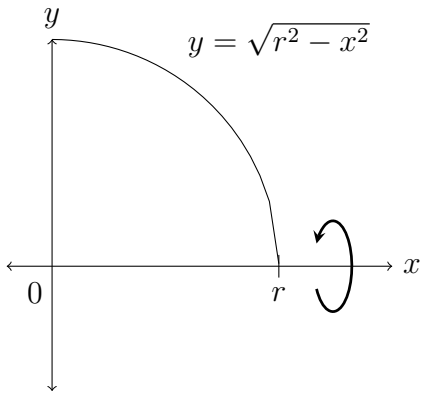
$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

1. **Example:** Find the surface area of a sphere of radius r.

   Thinking about the problem:

   What formula should I use to determine the surface area and why? Have I seen a problem similar to this one before? If so, which formula did I use?

   To determine the surface area of a sphere of radius r, I see that this is just twice the surface area of $y = \sqrt{r^2 - x^2}$ rotated about the x-axis from $0 \leq x \leq r$.



   So I will use the following formula for $y = \sqrt{r^2 - x^2}$ and then double my answer

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

Doing the problem:

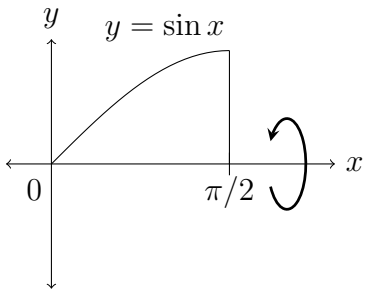
The problem asks to find the surface area of a sphere. This can be obtained by doubling the surface area found in rotating $y = \sqrt{r^2 - x^2}$ about the x-axis from $0 \leq x \leq r$. I find that $\dfrac{dy}{dx} = \dfrac{1}{2}(r^2 - x^2)^{-1/2} \cdot (-2x) = \dfrac{-x}{\sqrt{r^2 - x^2}}$. So by the formula for surface area,

$$\begin{aligned}
2S &= 2 \cdot \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \\
&= 2 \cdot \int_0^r 2\pi \sqrt{r^2 - x^2} \sqrt{1 + \left(\frac{-x}{\sqrt{r^2 - x^2}}\right)^2}\, dx \\
&= 4\pi \int_0^r \sqrt{r^2 - x^2} \sqrt{1 + \frac{x^2}{r^2 - x^2}}\, dx \\
&= 4\pi \int_0^r \sqrt{(r^2 - x^2) + \frac{(r^2 - x^2)x^2}{r^2 - x^2}}\, dx \\
&= 4\pi \int_0^r \sqrt{r^2 - x^2 + x^2}\, dx \\
&= 4\pi \int_0^r r\, dx \\
&= 4\pi \left(rx\right) \Big|_0^r \\
&= 4\pi r^2.
\end{aligned}$$

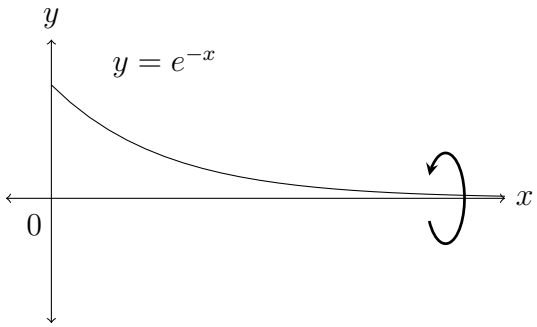
Therefore, the surface area of a sphere of radius r can be given by $4\pi r^2$.

Solutions should show all of your work, not just a single final answer.

2. Set up, but **do not evaluate**, a definite integral for the area of the surface formed by revolving the curve $y = \sin x$ for $0 \leq x \leq \pi/2$ around the x-axis.



3. Set up, but **do not evaluate**, an improper integral for the area of the surface formed by revolving the curve $y = e^{-x}$ for $x \geq 0$ around the x-axis.



4. T/F (with justification): Doubling the radius of a sphere will double the surface area of the sphere.