
7.2 Trigonometric Integrals

Trigonometric Identities and Formulas.

1. $\sin^2 \theta + \cos^2 \theta = 1$ 3. $\sin 2\theta = 2 \sin \theta \cos \theta$ 5. $\cos^2 \theta = \frac{1+\cos 2\theta}{2}$

2. $\tan^2 \theta + 1 = \sec^2 \theta$ 4. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ 6. $\sin^2 \theta = \frac{1-\cos 2\theta}{2}$

1. **Example:** Evaluate $\int_0^\pi \sin^3(5x) dx$.

Thinking about the problem:

What technique of integration should I use to evaluate the integral and why? Have I seen a problem similar to this one before? If so, which technique did I use?

I know that I need to evaluate a definite integral so my result should result in a number.

To determine which technique to use, I will focus on the integrand, i.e., $\sin^3(5x)$. In

this case, I cannot directly integrate $\int_0^\pi \sin^3(5x) dx$, so I will consider the Trigonometric

Identities. I will need to use an identity which involves powers of $\sin(x)$. I think I will

use the identity $\sin^2 \theta + \cos^2 \theta = 1$, so $\sin^2 \theta = 1 - \cos^2 \theta$. Then I can find $\sin^3(5x) =$

$\sin(5x) \sin^2(5x) = \sin(5x)(1 - \cos^2(5x))$. Since $\sin^3(5x) = \sin(5x)(1 - \cos^2(5x))$, I find

that $\int_0^\pi \sin^3(5x) dx = \int_0^\pi \sin(5x)(1 - \cos^2(5x)) dx$. If this new integral is more difficult

to solve than my original integral, then I probably made a mistake in my choice of the

identity to use. However, I see that I can integrate $\int_0^\pi \sin(5x)(1 - \cos^2(5x)) dx$ using

the technique of substitution, which is how I will proceed to evaluate the integral.

Doing the problem:

The problem asks to evaluate the integral. I note that I can use the identity $\sin^2 \theta + \cos^2 \theta = 1$ and find

$$\int_0^\pi \sin^3(5x) dx = \int_0^\pi \sin(5x)(1 - \cos^2(5x))dx.$$

Then I use the substitution $u = \cos(5x)$ so that $du = -5 \sin(5x)dx$. I also evaluate the bounds $\cos(5 \cdot 0) = 1$ and $\cos(5 \cdot \pi) = -1$ to find the new integral

$$\begin{aligned} \int_0^\pi \sin(5x)(1 - \cos^2(5x))dx &= \int_1^{-1} \sin(5x)(1 - u^2) \frac{du}{-5 \sin(5x)} \\ &= -\frac{1}{5} \int_1^{-1} 1 - u^2 du \\ &= \frac{1}{5} \int_{-1}^1 1 - u^2 du \\ &= \frac{1}{5} \left[u - \frac{u^3}{3} \right]_{-1}^1 \\ &= \frac{1}{5} \left(\left[1 - \frac{1}{3} \right] - \left[-1 - \frac{-1}{3} \right] \right) \\ &= \frac{1}{5} \left(2 - \frac{2}{3} \right) \\ &= \frac{4}{15}. \end{aligned}$$

Solutions should show all of your work, not just a single final answer.

2. Identify the trig identities to simplify the following integrands:

(a) $\int \sin^2 x \, dx$

(b) $\int \cos^2 x \, dx$

3. Evaluate $\int \sin^2 x \cos^2 x dx$. (*Hint*: Look at the exponents. Are they the same?)

(a) Which identity would you use to simplify the integrand?

(b) Using part (a), simplify the integrand.

(c) Evaluate $\int \sin^2 x \cos^2 x dx$ using the integrand found in part (b).

4. Evaluate $\int_0^\pi \cos^3 x \, dx$.

5. Evaluate $\int \cos x \sin^2 x \, dx$. (*Hint:* There may be more than one technique to evaluate this integral.)

(a) State the techniques you could use to evaluate the integral.

(b) Choose the most efficient technique and evaluate $\int \cos x \sin^2 x \, dx$.

6. T/F (with justification): The value of $\int_{-\pi}^{\pi} \sin^9 x \, dx$ is 0.