Math 1132 Worksheet 11.6 Name: ________________________ Discussion Section: _____

11.6 Absolute Convergence and the Ratio Test

Absolute Convergence. A series $\sum a_n$ is called absolutely convergent if the series of the absolute values $\sum |a_n|$ is convergent.

Note: If a series is absolutely convergent then it is also convergent.

Conditional Convergence. A series $\sum a_n$ is called conditionally convergent if it is convergent but not absolutely convergent.

The Ratio Test.

(i) If $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

(ii) If $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test is inconclusive; that is, no conclusion can be drawn about the convergence or divergence of $\displaystyle\sum_{n=1}^{\infty} a_n$.

Factorial.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \cdots \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1.$$

1. **Example:** Determine if the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}$$

Thinking about the problem:

Which test should I use to determine whether the series converges or diverges and why?

Have I seen a problem similar to this one before? If so, which test did I use?

To determine which test to use I will focus on the nth term, that is, $a_n = \frac{(2n)!}{(n!)^2}$. In this case I think I can use the Ratio Test because there are factorials in my nth term. To use the Ratio Test, I need to find $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$.

Doing the problem:

The problem asks whether the series is absolutely convergent, conditionally convergent, or divergent. I will apply the Ratio Test and note that $|a_n| = \frac{(2n)!}{(n!)^2}$ and $|a_{n+1}| = \frac{(2n+2)!}{((n+1)!)^2}$. So I see that

$$\begin{aligned}
\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n\to\infty} \left| \frac{\frac{(2n+2)!}{((n+1)!)^2}}{\frac{(2n)!}{(n!)^2}} \right| \\
&= \lim_{n\to\infty} \left| \frac{(2n+2)!(n!)(n!)}{(n+1)!(n+1)!(2n)!} \right| \\
&= \lim_{n\to\infty} \frac{n!}{(n+1)!} \cdot \frac{n!}{(n+1)!} \cdot \frac{(2n+2)!}{(2n)!} \\
&= \lim_{n\to\infty} \frac{1}{n+1} \cdot \frac{1}{n+1} \cdot (2n+2)(2n+1) \\
&= \lim_{n\to\infty} \frac{4n^2+6n+2}{n^2+2n+1} \\
&= \lim_{n\to\infty} \frac{4+6/n+2/n^2}{1+2/n+1/n^2} \\
&= 4 > 1.
\end{aligned}$$

Since $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 4 > 1$, the series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}$ is divergent.

Solutions should show all of your work, not just a single final answer.

2. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$$

(a) Which test or tests could you use to determine whether the series is absolutely convergent, conditionally convergent, or divergent?

(b) What happens if you apply the Ratio Test to the series?

(c) Is the Ratio Test conclusive?

(d) How would you test the series for absolute convergence, conditional convergence, or divergence?

3. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty}(-1)^n \frac{n^3}{4^n}$$

(a) Which test or tests could you use to determine whether the series is absolutely convergent, conditionally convergent, or divergent?

(b) Determine if the series is absolutely convergent, conditionally convergent, or divergent.

4. Determine if the series is absolutely convergent, conditionally convergent, or neither.

$$\sum_{n=1}^{\infty} \frac{n^2}{n!}$$

5. T/F (with justification)

   Convergence of a p-series for $p > 1$ can be shown with the ratio test.

6. T/F (with justification)

   There is an infinite series whose terms can be rearranged to be an infinite series with a different value.