Math 1132 Worksheet 11.5 Name: ________________________ Discussion Section: _____

11.5 Alternating Series

Alternating Series Test (AST). If the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \cdots \qquad b_n > 0$$

satisfies

(i) $b_{n+1} \leq b_n$ for all n

(ii) $\lim\limits_{n\to\infty} b_n = 0$

then the series is convergent.

Note: If either of the conditions of the AST are not met then you need to use a different test to determine whether the series is convergent or divergent.

Alternating Series Estimation Theorem. If $s = \sum(-1)^{n-1}b_n$ is the sum of an alternating series that satisfies

(i) $b_{n+1} \leq b_n$ for all n

(ii) $\lim\limits_{n\to\infty} b_n = 0$

then

$$|R_n| = |s - s_n| \leq b_{n+1}.$$

1. **Example:** Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$$

Thinking about the problem:

Which test should I use to determine whether the series converges or diverges and why? Have I seen a problem similar to this one before? If so, which test did I use?

To determine which test to use I write out a few terms of the series and notice that the sign of the successive terms of the series alternate between positive and negative. So I will use the AST. I will let $b_n = \frac{1}{2n+1}$. At this point I need to check the conditions for the Alternating Series Test.

Doing the problem:

The problem asks whether the series converges or diverges. I note that the terms of the series alternate between positive and negative and I let $b_n = \frac{1}{2n+1}$ and apply the Alternating Series Test (AST). Note that $b_n = \frac{1}{2n+1} > 0$ for all n. I can check that $b_{n+1} \leq b_n$ for all n by either checking this explicitly, or letting $f(x) = \frac{1}{2x+1}$ and checking that $f'(x) \leq 0$ for all $x \geq 1$. I choose to check $f'(x)$. I compute that $f'(x) = -\frac{2}{(2x+1)^2}$, which is negative for $x \geq 1$. Finally, $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{2n+1} = 0$.

So, by the AST, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$ converges.

Solutions should show all of your work, not just a single final answer.

2. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{\sqrt{n^3+2}}$$

(a) Which test or tests could you use to determine whether the series converges or diverges?

(b) State the test you would use to decide whether the series converges or diverges.

(c) State b_n.

3. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$$

4. Let s_n be the nth partial sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$, which converges by the Example given above.

   (a) Give a bound from above on $|s - s_{100}|$, as a decimal rounded to four digits after the decimal point.

   (b) How many terms (i.e., n) are needed to sum up in order that the sum is within 0.01 of the actual value of the series.

5. T/F (with justification)

   The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n}$ is alternating.

6. T/F (with justification)

   For the alternating series in problem 4, s lies between s_{100} and s_{101}.