Math 1132 Worksheet 11.2 Name: ______________________ Discussion Section: _____

11.2 Series

Series. Given a series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$, let s_n denote its nth partial sum:

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n.$$

If the sequence $\{s_n\}$ is convergent and $\lim_{n\to\infty} s_n = s$ exists as a real number, then the series $\sum_{n=1}^{\infty} a_n$ is called <u>convergent</u> and we write

$$a_1 + a_2 + \cdots + a_n + \cdots = s \text{ or } \sum_{n=1}^{\infty} a_n = s.$$

If the sequence $\{s_n\}$ is divergent, then the series is called <u>divergent</u>.

The Geometric Series. The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$$

is convergent if $|r| < 1$ and the sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \qquad |r| < 1.$$

If $|r| \geq 1$, the geometric series is divergent.

Test for Divergence. If $\lim_{n\to\infty} a_n$ does not exist or $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

1. **Example:** Compute the following series *exactly*. Write "divergent" if it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n$$

Thinking about the problem:

Which test should I use to determine whether the series converges or diverges and why? Have I seen a problem similar to this one before? If so, which test did I use?

To determine which test to use, I will focus on the nth term, that is, $a_n = \left(\frac{4}{11}\right)^n$. In this case, I think I will use what I know about geometric series because the nth term looks like a number to the nth power. Geometric series are written as $\sum_{n=1}^{\infty} ar^{n-1}$, so I will need to determine a and r in $\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n$. After I find a and r, I can follow through by checking if $|r| < 1$ and, if so, I can compute the value of $\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n$.

Doing the problem:

I see that

$$\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n = 1 + \left(\frac{4}{11}\right) + \left(\frac{4}{11}\right)^2 + \cdots$$

and

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$$

so I can compare each term to find that $a = 1$ and $r = \frac{4}{11}$. Now I know that $\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)^n$ is a geometric series, so I can use what I know about the convergence of geometric series.

Since $|r| = \left|\frac{4}{11}\right| < 1$, I know that $\sum_{n=0}^{\infty}\left(\frac{4}{11}\right)^n$ converges and converges to

$$\frac{a}{1-r} = \frac{1}{1-4/11} = \frac{11}{7}.$$

Solutions should show all of your work, not just a single final answer.

2. Compute the following series *exactly*. Write "divergent" if it diverges.

$$\sum_{n=1}^{\infty} \frac{7^n}{4^{n+3}}$$

(a) Which test or tests could you use to determine whether the series converges or diverges?

(b) State the test you would use to decide whether the series converges or diverges.

(c) Write out the first 4 (four) terms of $\sum_{n=1}^{\infty} \frac{7^n}{4^{n+3}}$. Write out the first 4 (four) terms of $\sum_{n=1}^{\infty} ar^{n-1}$. Compare terms to find an a and r so that $\sum_{n=1}^{\infty} \frac{7^n}{4^{n+3}} = \sum_{n=1}^{\infty} ar^{n-1}$.

3. Use geometric series to convert the repeating decimal $.9\overline{34} = .934343434\ldots$ into a fraction in reduced form.

   (a) Can you write $.9\overline{34} = .934343434\ldots$ as a geometric series?

   (b) If $.9\overline{34} = .934343434\ldots = \sum_{n=1}^{\infty} ar^{n-1}$, what is a?

   (c) If $.9\overline{34} = .934343434\ldots = \sum_{n=1}^{\infty} ar^{n-1}$, what is r? Is $|r| < 1$?

   (d) Use the geometric series found in (a) through (c) to convert $.9\overline{34} = .934343434\ldots$ into a fraction.

4. For the series $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+3}\right)$, determine a formula for its Nth partial sum. If the series converges, determine its value exactly.

   (a) Write the first 5 (five) terms of $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+3}\right)$.

   (b) Determine a formula for the Nth partial sum of $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+3}\right)$.

   (c) Take the limit of the Nth partial sum of $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+3}\right)$ as $N \to \infty$.

5. T/F (with justification): If $a_n \to 0$ as $n \to \infty$ then the series $\sum_{n=1}^{\infty} a_n$ converges.

6. T/F (with justification): The convergence of a series $\sum_{n=1}^{\infty} a_n$ is unaffected by dropping its first few terms.