
10.3 Polar Coordinates

Polar Coordinates/Cartesian Coordinates.

1. A) To find the Cartesian coordinates of a point when the polar coordinates are known, use:

$$x = r \cos \theta \quad y = r \sin \theta$$

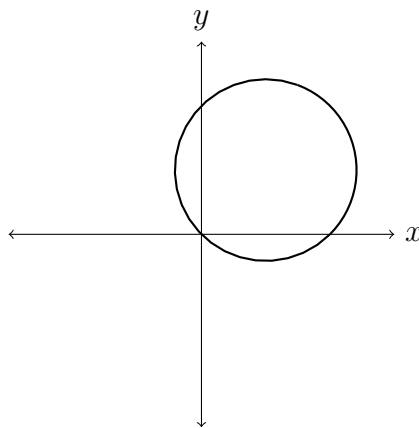
2. B) To find the polar coordinates of a point when the Cartesian coordinates are known, use:

$$r^2 = x^2 + y^2 \quad \tan \theta = \frac{y}{x}$$

Derivatives.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\left(\frac{dr}{d\theta} \cdot \sin \theta\right) + r \cos \theta}{\left(\frac{dr}{d\theta} \cdot \cos \theta\right) - r \sin \theta}$$

1. **Example:** Here is the plot of the polar equation $r = \sin \theta + \cos \theta$.



Fill in the table below and find the equation of the tangent line to the curve at $(x, y) = (0, 0)$.

θ	$\sin \theta + \cos \theta$	(r, θ)	(x, y)
0			
$\pi/2$			
π			
$3\pi/2$			

Thinking about the problem:

How should I approach this problem? What is it asking me to do? Have I seen a problem similar to this one before? If so, what did I do?

The first part of the question is asking me to fill in a table for values of $r = \sin \theta + \cos \theta$, (r, θ) , and (x, y) . Using the formulas $x = r \cos \theta$ and $y = r \sin \theta$, I can compute the values in the table. The second part of the question is asking me to find the equation of the tangent line to the curve at $(x, y) = (0, 0)$. First, I will need to find for what value θ does $(x, y) = (0, 0)$. Then I can find the derivative using the formula

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\left(\frac{dr}{d\theta} \cdot \sin \theta\right) + r \cos \theta}{\left(\frac{dr}{d\theta} \cdot \cos \theta\right) - r \sin \theta}.$$

Finally, I will recall that the equation of a tangent line at the point (x_0, y_0) is

$$y - y_0 = \frac{dy}{dx}(x - x_0).$$

I can then find the information I need to find the equation of the tangent line.

Doing the problem:

I can compute the values in the table using the formula $x = r \cos \theta$ and $y = r \sin \theta$ to find the following:

θ	$\sin \theta + \cos \theta$	(r, θ)	(x, y)
0	1	(1, 0)	(1, 0)
$\pi/2$	1	(1, $\pi/2$)	(0, 1)
π	-1	(-1, π)	(-1, 0)
$3\pi/2$	-1	(-1, $3\pi/2$)	(0, -1)

I find that $(x, y) = (0, 0)$ is when $r = \sin \theta + \cos \theta = 0$ so $\sin \theta = -\cos \theta$, which is true when $\theta = \frac{\pi}{4}$. To find the equation of the tangent line, I find

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \\ &= \frac{\left(\frac{dr}{d\theta} \cdot \sin \theta\right) + r \cos \theta}{\left(\frac{dr}{d\theta} \cdot \cos \theta\right) - r \sin \theta} \end{aligned}$$

In my formula above I need to find $\frac{dr}{d\theta}$. Since $r = \sin \theta + \cos \theta$ then $\frac{dr}{d\theta} = \frac{d}{d\theta}(\sin \theta + \cos \theta) = \cos \theta - \sin \theta$.

$$\begin{aligned} \frac{dy}{dx} &= \frac{(\cos \theta - \sin \theta) \cdot \sin \theta + (\sin \theta + \cos \theta) \cdot \cos \theta}{(\cos \theta - \sin \theta) \cdot \cos \theta - (\sin \theta + \cos \theta) \cdot \sin \theta} \\ &= \frac{\cos \theta \cdot \sin \theta - \sin^2 \theta + \sin \theta \cdot \cos \theta + \cos^2 \theta}{\cos^2 \theta - \sin \theta \cdot \cos \theta - \sin^2 \theta - \cos \theta \cdot \sin \theta} \\ &= \frac{\cos^2 \theta - \sin^2 \theta + 2 \cos \theta \sin \theta}{\cos^2 \theta - \sin^2 \theta - 2 \sin \theta \cos \theta} \end{aligned}$$

Letting $\theta = \frac{\pi}{4}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\cos^2\left(\frac{\pi}{4}\right) - \sin^2\left(\frac{\pi}{4}\right) + 2\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}{\cos^2\left(\frac{\pi}{4}\right) - \sin^2\left(\frac{\pi}{4}\right) - 2\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)} \\ &= \frac{\frac{1}{2} - \frac{1}{2} + 2 \cdot \frac{1}{2}}{\frac{1}{2} - \frac{1}{2} - 2 \cdot \frac{1}{2}} \\ &= -1 \end{aligned}$$

So I can conclude that the tangent line is

$$y - y_0 = \frac{dy}{dx}(x - x_0)$$

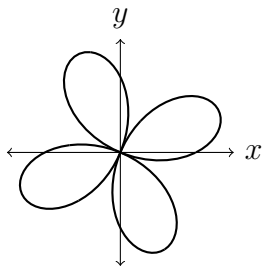
$$y = -x.$$

Solutions should show all of your work, not just a single final answer.

2. Give two additional representations of P in polar coordinates:

$$(a) P = \left(2, \frac{\pi}{4}\right), \quad (b) P = \left(-1, -\frac{\pi}{3}\right).$$

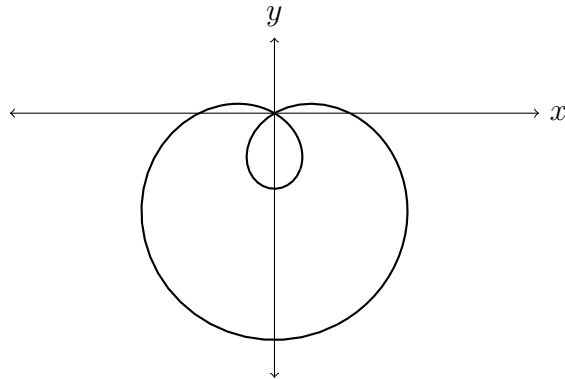
3. Below is a graph of the polar equation $r = \sin(2\theta) + \cos(2\theta)$. Fill in the table below for this polar graph.



θ	$\sin(2\theta) + \cos(2\theta)$	(r, θ)	(x, y)
0			
$\pi/2$			
π			
$3\pi/2$			
2π			

Based on this table, and additional data if it seems needed, draw arrows on the curve (including on each loop) to indicate the direction of θ as it increases from 0 to 2π .

4. Below is a plot of the polar equation $r = 1 - 2 \sin \theta$.



Find the equation of the tangent line to the curve $r = 1 - 2 \sin \theta$ at $(x, y) = (1, 0)$.

5. T/F (with justification)

Every point in the plane besides the origin can be written in polar coordinates (r, θ) with $r < 0$.