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## 10.1 Curves Defined by Parametric Equations

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1. **Example:** In the following parametric equations, eliminate the parameter to obtain a single equation only in terms of  $x$  and  $y$ . Then describe the graph it traces.

$$x = -\sin(2t), \quad y = \cos(2t); \quad 0 \leq t \leq 2\pi.$$

*Thinking about the problem:*

How should I eliminate the parameter  $t$ ? Have I seen a problem similar to this one before? If so, what approach did I use?

I notice that since  $x$  and  $y$  look like parametric equations of a circle, I should expect the single equation in terms of  $x$  and  $y$  to look like a circle. To describe the graph, I will see my resulting equation in terms of  $x$  and  $y$  to find the graph it traces, but use the individual  $x$  and  $y$  to find the orientation and starting point of my parametrization.

*Doing the problem:*

I know that  $\sin^2(2t) + \cos^2(2t) = (-\sin(2t))^2 + (\cos(2t))^2 = 1$ , so replacing  $x = -\sin(2t)$  and  $y = \cos(2t)$ , I find the resulting equation  $x^2 + y^2 = 1$  and the equation is only in terms of  $x$  and  $y$ . So I know my graph will be a circle of radius 1 centered at the origin. I know that since  $x = -\sin(2 \cdot 0) = 0$  and  $y = \cos(2 \cdot 0) = 1$ , the starting point of my parametrization is  $(0, 1)$ . Similarly, I know that  $x = -\sin(2t)$  starts out negative whereas  $y = \cos(2t)$  starts positive, so my orientation will be  $x$  starting at 0 and going negative (left) and  $y$  starting at 1 and staying positive. Therefore my parametrization is counterclockwise. Finally, since  $0 \leq t \leq 2\pi$ , I see that my parametrization traces the circle twice, since  $x = -\sin(2t)$  and  $y = \cos(2t)$  indicates my parametrization goes twice the speed as  $\sin(t)$  and  $\cos(t)$ , which would trace the graph once in  $0 \leq t \leq 2\pi$ .

**Solutions should show all of your work, not just a single final answer.**

2. In the following parametric equations, eliminate the parameter to obtain a single equation only in terms of  $x$  and  $y$ .

$$x = t^2 + 4, \quad y = 3t^2; \quad 0 \leq t \leq 2.$$

(a) Write  $t$  in terms of  $x$ .

(b) Write  $t$  in terms of  $y$ .

(c) Set your answer to (a) and (b) equal to each other.

3. In the following parametric equations, eliminate the parameter to obtain a single equation only in terms of  $x$  and  $y$ .

$$x = 3 \cos t, \quad y = 3 \sin t; \quad 0 \leq t \leq \frac{\pi}{2}.$$

4. Find a parameterization of the following: a circle centered at the origin with radius 16, oriented counterclockwise with initial point  $(0, 16)$ .

(a) What are the parametric equations for a circle?

(b) Alter the parametric equations from (a) to trace a circle with radius 16.

(c) Alter the parametric equations from (b) to be oriented counterclockwise.

(d) Alter the parametric equation from (c) so when  $t = 0$ ,  $x = 0$  and  $y = 16$ .

5. Find a parameterization of the following: a circle centered at  $(-2, -3)$  with radius 8, oriented clockwise with initial point  $(6, -3)$ .

6. T/F (with justification)

The parametric curve  $(\sin t, -\cos t)$  as  $t$  increases traces out a circle counterclockwise.