Math 1132 Worksheet 7.2 Name: ________________________ Discussion Section: _____

7.2 Trigonometric Integrals

Trigonometric Identities and Formulas.

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\tan^2\theta + 1 = \sec^2\theta$
3. $\sin 2\theta = 2\sin\theta\cos\theta$
4. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
5. $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$
6. $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

Example: Evaluate $\displaystyle\int_0^\pi \sin^3(5x)\,dx$.

Thinking about the problem:

To integrate a power like $\sin^3(5x)$, let's write $\sin^3\theta$ in terms of lower powers. By the first trigonometric identity above, we can write $\sin^2\theta = 1-\cos^2\theta$, so

$$\sin^3\theta = \sin^2\theta\sin\theta = (1-\cos^2\theta)\sin\theta.$$

Therefore (using $\theta = 5x$)

$$\int_0^\pi \sin^3(5x)\,dx = \int_0^\pi (1-\cos^2(5x))\sin(5x)\,dx.$$

Doing the problem:

After rewriting of the function being integrated, let's use the substitution $u = \cos(5x)$, so $du = -5\sin(5x)\,dx$:

$$\int (1-\cos^2(5x))\sin(5x)\,dx = \int (1-u^2)\frac{-du}{5} = -\frac{1}{5}\int (1-u^2)\,du.$$

Let's turn x-bounds into u-bounds in the definite integral:

$$x = 0 \Longrightarrow u = \cos(5\cdot 0) = \cos 0 = 1, \quad x = \pi \Longrightarrow u = \cos(5\pi) = -1.$$

Therefore

$$\begin{aligned}
\int_0^\pi \sin^3(5x)\,dx &= \int_{x=0}^{x=\pi} (1-\cos^2(5x))\sin(5x)\,dx \\
&= -\frac{1}{5}\int_{u=1}^{u=-1} (1-u^2)\,du \quad \text{(Note the order of integration)} \\
&= \frac{1}{5}\int_{-1}^{1} (1-u^2)\,du \quad \text{(Sign change in the order of integration)} \\
&= \frac{1}{5}\left(u-\frac{u^3}{3}\right)\Big|_{-1}^{1} \\
&= \frac{1}{5}\left(\left(1-\frac{1}{3}\right)-\left(-1-\frac{-1}{3}\right)\right) \\
&= \frac{1}{5}\left(1-\frac{1}{3}+1-\frac{1}{3}\right) \\
&= \frac{1}{5}\left(2-\frac{2}{3}\right) \\
&= \frac{4}{15}.
\end{aligned}$$

Solutions should show all of your work, not just a single final answer.

1. Show the formulas $\sin(2x) = 2\sin x\cos x$ and $\cos(2x) = \cos^2 x - \sin^2 x$ each imply the other one using differentiation: differentiate each identity and simplify to turn the first formula into the second and the second formula into the first.

2. Identify the trigonometric identities to simplify the following integrands, and carry out the integration. Remember to include $+C$ in the final answer.

   (a) $\displaystyle\int \sin^2 x\,dx$

   (b) $\displaystyle\int \cos^2 x\,dx$

3. Evaluate $\int \sin^2 x \cos^2 x \, dx$ by using a trigonometric identity involving $\sin x \cos x$ to simplify the integrand.

4. Evaluate the definite integral $\int_0^{\pi} \sin^3 x \, dx$.

5. Evaluate $\int \cos x \sin^2 x \, dx$. (There may be more than one technique that works.)

6. T/F (with justification): The value of $\int_{-\pi}^{\pi} \sin^9 x \, dx$ is 0.