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## Integration by Partial Fractions

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**Solutions should show all of your work, not just a single final answer.**

**Example.** Compute  $\int \frac{2x+1}{x^2-4} dx$  using partial fractions.

**Solution.** Write  $\frac{2x+1}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$  for some  $A$  and  $B$ . Clearing the denominator,

$$2x+1 = A\frac{x^2-4}{x+2} + B\frac{x^2-4}{x-2} = A(x-2) + B(x+2) = (A+B)x + (-2A+2B).$$

Here are two ways to find  $A$  and  $B$  (you only need to use one method).

Method 1. Equating coefficients of common terms on both sides,  $A+B=2$  and  $-2A+2B=1$ , or

$$\begin{aligned} A+B &= 2 \\ A-B &= -\frac{1}{2}. \end{aligned}$$

Adding and subtracting the equations,  $2A=3/2$  and  $2B=5/2$ , so  $A=3/4$  and  $B=5/4$ .

Method 2. Setting  $x=2$  and  $x=-2$  shows  $5=4B$  and  $-3=-4A$ , so  $A=3/4$  and  $B=5/4$ .

Either way,  $\frac{2x+1}{x^2-4} = \frac{3/4}{x+2} + \frac{5/4}{x-2}$ , so

$$\int \frac{2x+1}{x^2-4} dx = \int \left( \frac{3/4}{x+2} + \frac{5/4}{x-2} \right) dx = \frac{3}{4} \ln|x+2| + \frac{5}{4} \ln|x-2| + C.$$

1. Compute  $\int \frac{dx}{x^2-4x+3}$  using partial fractions.

2. Compute  $\int \frac{x+1}{x^3-x^2-6x} dx$  using partial fractions.

3. Compute  $\int \frac{x^2+x+1}{x(x^2+4)} dx$  using partial fractions.

4. Compute  $\int \frac{dx}{(x-a)(x-b)}$  using partial fractions when  $a \neq b$ . Express the final answer in terms of  $a$  and  $b$ .

5. T/F (with justification) Computing  $\int \frac{x}{x^2-1} dx$  requires partial fractions.