Integration by Parts

Solutions should show all of your work, not just a single final answer.

Do each problem as follows: (1) specify u and dv, (2) compute du and v, (3) use integration by parts with your choice of u and dv, (4) if you need integration by parts more than once, each time go through steps 1, 2, and 3 again.

Example. Compute $\int x^2 e^x \, dx$ using integration by parts.

Solution.

(1) Set $u = x^2$ and $dv = e^x \, dx$.

(2) We have $du = 2x \, dx$ and $v = e^x$.

(3) Now $\int x^2 e^x \, dx = \int u \, dv = uv - \int v \, du = x^2 e^x - \int e^x (2x) \, dx = x^2 e^x - 2 \int x e^x \, dx$.

(4) To find $\int x e^x \, dx$, set $u = x$ and $dv = e^x \, dx$, so $du = dx$ and $v = e^x$. Then $\int x e^x \, dx = \int u \, dv = uv - \int v \, du = x e^x - \int e^x \, dx = x e^x - e^x$.

(5) Substituting the result of (4) into (3),

$$\int x^2 e^x \, dx = x^2 e^x - 2(x e^x - e^x) + C = (x^2 - 2x + 2) e^x + C.$$

1. Compute $\int x \cos(5x) \, dx$ using integration by parts.

2. Compute $\int x^2 e^{3x} \, dx$ using integration by parts.

3. Compute $\int_0^{\pi} x^2 \sin x \, dx$ using integration by parts.

4. Show $\int_0^{2\pi} \sin(mx) \cos(nx) \, dx = 0$ for all positive integers m and n using integration by parts. This is *fundamental* in the mathematical analysis of vibrations. (Hint: Treat $m \neq n$ and $m = n$ separately. It may help to try it first for specific m and n, such as $m = 2$ and $n = 3$, and $m = 5$ and $n = 5$, but the solution should treat the general case.)

5. T/F (with justification): For differentiable $f(x)$, $\int_0^{\pi} f(x) \cos x \, dx = -\int_0^{\pi} f'(x) \sin x \, dx$.