Representations of Functions as Power Series

Solutions should show all of your work, not just a single final answer.

1. Use the power series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ when $|x| < 1$ to find a power series centered at $x = 0$ for the following related functions. Find the interval of convergence in all cases.
   (a) $\frac{1}{2-5x}$ Hint: Write $\frac{1}{2-5x} = \frac{1}{2} \cdot \frac{1}{1-?}$.

   (b) $\frac{1}{1+x^4}$

   (c) $\frac{1}{(1-x)^3}$ Hint: Differentiate twice.

2. Use the power series $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$ and its radius of convergence 1 to find a power series centered at $x = 0$ for $\ln(1-9x)$ and its radius convergence.

3. Use power series to estimate $\int_0^{1/2} \frac{dx}{1+x^4}$ to within .00001 by the following steps.
   (a) Express $\int \frac{dx}{1+x^4}$ as a power series, starting with the power series you found in 1b.
   (b) Find the radius of convergence of the power series in part a.
   (c) Use the previous parts and the Alternating Series Estimation Theorem to estimate $\int_0^{1/2} \frac{dx}{1+x^4}$ to within .00001. Round your *estimate* to 5 digits.

4. T/F (with justification)
   If $\sum_{n=0}^{\infty} c_n x^n$ has radius of convergence 3 then $\sum_{n=0}^{\infty} c_n x^{2n}$ has radius of convergence 9.